A Modified Approach To Calculate The Stock Price That Has Non-Constant Growth Rates In Dividends: A Teaching Note

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ABSTRACT

The traditional approach to estimate the stock price with nonconstant growth rates is to calculate the dividend payments under the nonconstant growth rate periods and the stockt price at a point where the growth rate becomes constant by using the constant growth rate model. The stock price equals to the present value of those dividend payments and the stock price at the point where the growth rate starts becoming constant. This paper modifies the analysis and provides an improved and easier approach to estimate the price with the nonconstant growth rates.

INTRODUCTION

The dividend discounted model states that the value of the financial asset equals to the present value of all future dividend payments. Under this model, there are three scenarios where the future dividend payments can be estimated.

1. If the future dividend payments are constant, i.e., the growth rate in dividend payments is constant; under this scenario the present value of the future dividend payments P0

$$P_0 = D/(1+K) + D/(1+K)^2 + D/(1+K)^3 + \dots\dots\dots\dots$$

Where: P_0 is the stock price today
D is the constant dividend payment
K is the discount rate

The equation becomes

$$P_0 = D/K$$

2. The future dividend payments will grow at a constant growth rate g. Under this scenario the present value of the future dividend payments

$$P_0 = D_1/(1+K)^1 + D_2\,/(1+K)^2 + D_3/\,(1+K)^3 + \dots\dots\dots$$

$$P_0 = D_1/(1+K)^1 + D_1(1+g)/ (1+K)^2 + D_1*(1+g)^2/(1+K)^3$$

The equation becomes

$$P_0 = D_1/(K-g)$$

where

P_0 is the price of the financial asset today,
g is the constant growth rate
D_1 is the expected dividend payment received in one year,
K is the discount rate of the firm.

3. The third scenario is where the dividend payments do not grow at a constant growth rate. This is particularly true for many technology, small, and growing firms. However, the assumption is that as the company goes through the life cycle and at some point in the future, the growth rate will become constant.

To price the financial asset, it is needed to estimate the nonconstant growth dividends and the point where the constant rate will begin. The price of the financial asset equals to the present value of the future cash flows, including the future dividend payments and the capital gain.

The traditional approach to calculating the price of the asset is to calculate the dividend payments under the nonconstant growth rate periods and the asset price at a point where the growth rate becomes constant by using the constant growth rate model.

This paper modifies the analysis and provides an improved and easier approach to estimating the price under the nonconstant growth scenario. Instead of calculating the future asset price at a point where the growth rate becomes constant, the modified approach suggests to calculate the price one year before the growth rate becomes constant by using the constant growth model. For example, if the growth rate is nonconstant for the following two years, the growth rate then will grow at a constant growth rate. The traditional approach is to calculate the nonconstant growth dividends, D_1 and D_2. Additionally the traditional approach calculates the asset price at a point where the growth rate becomes constant, P_2. Adding the present values of D_1, D_2 and P_2 will be the asset price today, P_0.

The modified approach suggests that one only needs to calculate the nonconstant dividend payments D_1 and D_2, then calculate the asset price one period before the growth rate becomes constant P_1. Adding the present values of D_1 and P_1 will be the asset price today P_0.

The modified approach has some unique advantages: it does not require to calculate the first dividend payment after the growth rate becomes constant (in this case, it is D_3). Additionally it does not require to calculate the asset price at the point where the growth rate becomes constant (in this case it is P_2). It only requires to calculate the nonconstant dividend payments (D_1 and D_2) and the price at a point where the growth rate becomes constant P_1.

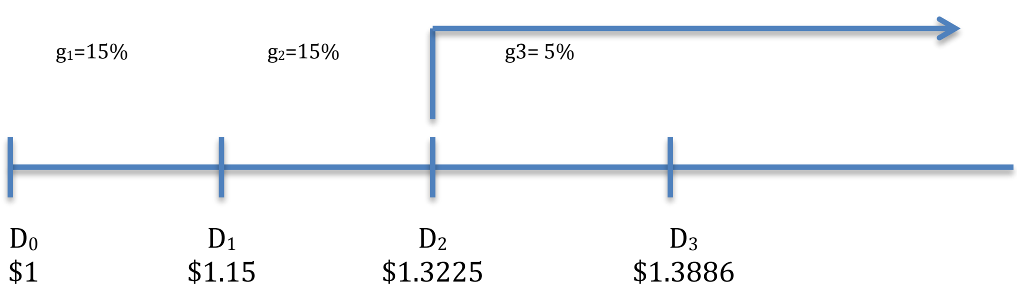
This modified approach simplifies the calculation by not needing to calculate two extra variables (D_3 and P_2)

The following example demonstrates how the modified approach works.

- Last year's dividend = \$1.00
- Dividend will grow at 15% for two years
- Dividend will grow at a constant growth rate 5% after two years
- Required return K is 11%

Calculate the price of the asset today P_0

Solution #1, traditional approach: We calculate the nonconstant growth dividends, D_1 and D_2. We further calculate the price in year 2 (after year two growth rate becomes constant) by using the constant growth model $D_3/(K-g)$. We then add the present values of the D_1, D_2 and P_2 to derive the value of the asset in year 0, P_0= \$20.893



D_0	D_1	D_2	D_3
1	\$1.15 {=D0 (1+15%)}	\$1.3225 {=D1 (1+15%)}	\$1.3886 {=D2 (1+5%}
$\mathbf{P_0 = [\frac{D_1}{(1+K)^1} + \frac{D_2}{(1+K)^2} + \frac{P_2}{(1+K)^2}]}$		$\mathbf{P_2 = \frac{D_3}{(k-g)}}$	
P_0 =\$20.8930 P_0 = [\$1.15/(1+11%)^1 + [(1.3225+23.1433)/(1+11%)^2		P_2= \$23.1433 P_2= \$1.3886/(11%-5%)	

Solution #2, the modified approach: we calculate the price of the asset one period earlier than when the growth rate becomes constant. In this case, we calculate the price in year one P_1 by using $D_2/(K-g)$. We then add the present value of D_1 and P_1 and get the asset price today P_0 = \$20.893

According to the Gordon constant growth model[1]:

$$P_0 = \frac{D_0(1+g)}{K-g} = \frac{D_1}{K-g}$$

Therefore, given the scenario, we can use D_2 to calculate P_1 at the constant growth rate

$$P_1 = \frac{D_2}{(K-g)}$$

This calculation is simplified one step of discounting the dividends from the year three to infinitive (P_2) under a constant growth rate, compared to the traditional Gordon growth rate calculation.

In fact, the total present value of two discounted dividends D_2 and P_2 is equal to the present value of only the discounted year two dividend (D_2) to year one (P_1).

Proof: according to the dividend discount model, the asset price today equals to the present value of future cash flows, including dividends and capital gain. The asset price in year one should equal to the present values of D_2 and P_2.

$$P_1 = \frac{D_2}{(1+K)^1} + \frac{P_2}{(1+K)^1} \qquad (1)$$

However,

$$P_2 = \frac{D_3}{(K-g)} = \frac{D_2(1+g)}{(K-g)}$$

So, replace P_2 to (1), we have:

$$P_1 = \frac{D_2}{(1+K)^1} + \frac{D_2\,(1+g)}{(K-g)(1+K)^1}$$

$$= \frac{D_2(K-g)+D_2\,(1+g)}{(1+K)(K-g)}$$

D_0	D_1	D_2	D_3
1	\$1.15	\$1.3225	\$1.3886
$\mathbf{P_0 = \frac{D_1}{(1+K)^1} + \frac{P_1}{(1+K)^1}}$	$\mathbf{P_1 = \frac{D_2}{K-g}}$		
P_0 = \$20.8933 P_0 = [\$1.15/(1+11%)^1 +[22.0416/(1+11%)^1	P_1= \$22.0416 P_1= \$1.3225/[11%-5%)		

$$= \frac{D_2\,(K-g+1+g)}{(1+K)(K-g)}$$

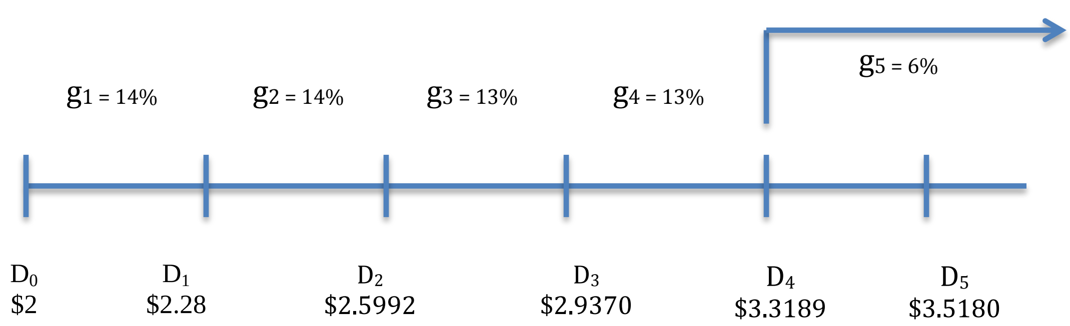
$$= \frac{D_2}{(K-g)}$$

According to our mathematical derivation, we can calculate the asset price at a point one year before the growth rate becomes constant by using the constant growth model.

Example 2:

- Last year's dividend = \$2.00
- Dividend will grow at 14% for first two years
- Dividend will grow at 13% for the following two years
- Dividend will grow at a constant growth rate 6% after four years
- Required return K is 12%

Calculate the price of the asset today P_0



Solution 1					
D_0	D_1	D_2	D_3	D_4	D_5
\$2	\$2.28	\$2.5992	\$2.9370	\$3.3189	\$3.5180
$\boldsymbol{P_0 = \frac{D_1}{(1+K))^1} + \frac{D_2}{(1+K))^2} + \frac{D_3}{(1+K)^3} + \frac{D_4 + P_4}{(1+K)^4}}$				$\boldsymbol{P_4 = \frac{D_5}{K-g}}$	
P_0 = \$45.5679 $P_0 = \frac{\$2.28}{1.12^1} + \frac{\$2.5992}{1.12^2} + \frac{2.9370}{1.12^3} + \frac{(3.3189 + \$58.63)}{1.12^4}$				P_4 = \$58.63 $P_4 = \frac{\$3.5180}{12\% - 6\%}$	

Solution 2:					
D_0	D_1	D_2	D_3	D_4	D_5
\$2	\$2.28	\$2.5992	\$2.9370	\$3.3189	\$3.5180
$\boldsymbol{P_0 = \frac{D_1}{(1+K))^1} + \frac{D_2}{(1+K))^2} + \frac{D_3 + P_3}{(1+K)^3}}$			$\boldsymbol{P_3 = \frac{D_4}{k-g}}$		
P_0 = \$45.5704 $P_0 = \frac{\$2.28}{1.12^1} + \frac{\$2.5992}{1.12^2} + \frac{(2.9370 + \$55.315)}{1.12^3}$			P_3 = \$55.315 $P_3 = \frac{\$3.3189}{12\% - 6\%}$