

Guesstimation as a Pedagogical Device in Finance

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Abstract

This paper discusses how guesstimation is used as a pedagogical device in a quantitative finance course. A guesstimate is a quick and intuitive approximation to a mathematic problem. Many financial formulas, such as Black-Scholes, provide little intuition and are often miscalculated by students. Simple algebraic errors cause students to make gross miscalculation in the magnitude of the value or even its sign. Introducing these complex topics with guesstimation provides students with insight into the problem and a reasonable benchmark to assess the exact solution. This paper will discuss how guesstimation is used in a financial derivatives course to introduce complex topics and develop problem solving strategies for a lifetime.

INTRODUCTION

A guesstimate is a quick and intuitive approximation to a mathematic problem that can be used to enhance student learning. Guesstimates provide intuition into exact formulas which offer little intuition and are often grossly miscalculated. Guesstimation can also be used as a test-taking strategy in which students use the guesstimate as a benchmark to evaluate the reasonableness of their exact solution. Students can also use guesstimation to answer technical interview questions. Lastly, guesstimation is a valuable skill for financial managers that must make quick estimates and judgments.

The guesstimation procedures presented in this paper are simple formulas that students could remember and use quickly without the aid of a calculator, computer, or statistical table. The guesstimation formulas include essential parameters of the problem and provide insight into the relationships between key parameters and the solution. The guesstimates are also shown to provide reasonable solutions for a range of real world parameter values.

The next several sections propose guesstimates for the calculation of option prices, bond duration and convexity, swap prices, and currency premiums. Each guesstimate is introduced with a problem that requires an exact solution. The guesstimate equation is presented with insight into the nature of the underlying relationships. The guesstimate is then compared with the exact solution for a range of reasonable parameter values. The last sections suggest how guesstimation can be used in the classroom and enhanced with the use of key market benchmarks.

OPTION PRICE GUESSTIMATION

Call options are contracts that give the owner the right, but not the obligation, to buy an asset at specified price and time. The models commonly used to value options include the Black-Scholes model, the binomial model, and Monte Carlo simulation. The option pricing models are complex and are often miscalculated by students. A guesstimate could be used to introduce the topic of option valuation and also provide students with a quick and intuitive estimate. Finance recruiters have also been known to ask students to compute options prices without the aid of calculators (Crack, 2007).

The guesstimate for at-the-money call options is

$$\tilde{C} = .4S(r - \delta + \sigma) \tag{1}$$

Where

- C = call option value
- S = current stock price (S is assumed to be equal to K, the strike price)
- δ = dividend yield (annualized)
- r = risk-free rate or return (annualized)
- σ = standard deviation of stock returns (annualized)

The call option guesstimate can be interpreted as the present value of the call option's expected payoff. The payoff of a call option is $S_T - K$ at maturity if the stock price is above the strike price or the payoff is zero if the price is below the strike. If stock returns were certain, the index would appreciate by the risk-free rate minus the value of the paid dividends, which is approximated by $r - \delta$. Standard deviation is a measure of the uncertainty of the stock returns. The guesstimate assumes risky asset prices are expected to increase by $r - \delta$ but can be higher or lower by one standard deviation ($\pm 1\sigma$). If the probability of a $+1\sigma$ increase is 50%, then the payoff at maturity is $.5S(r - \delta + \sigma)$. The present value of the expected payoff for an at-the-money option is approximated by the guesstimate of $.4S(r - \delta + \sigma)$.

Call Option Sample Problem: What is the value of a one-year, at-the-money call option on the S&P 500 index? Assume that the S&P 500 index is \$1,000, the dividend yield is 2%, the index has a standard deviation of 20%, and the one year Treasury yield is 4%.

The guesstimate is:

$$\tilde{C} = .4 \times 1000(.04 - .02 + .2) = \$88.00$$

The Black-Scholes call option formula (McDonald, 2006) is:

$$C = Se^{-\delta t} N(d_1) - Ke^{-rt} N(d_2) \tag{2}$$

where

$$d_1 = \frac{\ln(S/K) + \left(r - \delta + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}} \quad (3)$$

$$d_2 = d_1 - \sigma\sqrt{t} \quad (4)$$

S = current stock price
 K = option strike price
 δ = continuously compounded dividend yield (annualized)
 r = continuously compounded risk-free rate or return (annualized)
 σ = standard deviation of stock returns (annualized)
 T = option maturity in years
 N(x) = cumulative normal distribution function

The Black-Scholes solution to the sample problem is:

$$C = 1,000e^{-.02 \times 1} 0.57926 - 1,000e^{-.04 \times 1} 0.50 = 87.39$$

where

$$d_1 = \frac{\ln(1,000/1,000) + \left(.04 - .02 + \frac{1}{2} \cdot .2^2\right)1}{.2\sqrt{1}} = .2 \quad \text{and} \quad d_2 = .2 - .2\sqrt{1} = 0$$

$$N(d_1) = .57926 \quad \text{and} \quad N(d_2) = 0.50$$

The call option guesstimate is reasonably accurate for varying levels for σ , r, and δ . Table 1 compares the guesstimate with the exact Black-Scholes solution. The guesstimate errors are between -2% and 5% of the Black-Scholes price using standard deviations ranging from 10% to 80%. Table 2 shows that the guesstimate errors are between 4% and -6% using risk-free rates ranging from 0% to 10%. Table 3 shows that the guesstimate errors are between -3% and 13% using dividend yields ranging from 0% to 10%. The bolded row in each table shows the sample problem solution.

Table 1. Call Option Guesstimate and Exact Solution Comparison for different levels of σ

σ	Exact	Guesstimate	Error %
10%	\$ 49.18	\$ 48.00	-2%
20%	\$ 87.39	\$ 88.00	1%
30%	\$ 125.68	\$ 128.00	2%
40%	\$ 163.74	\$ 168.00	3%
60%	\$ 238.69	\$ 248.00	4%
80%	\$ 311.47	\$ 328.00	5%

Table 2. Call Option Guesstimate and Exact Solution Comparison for different levels of r

r	Exact	Guesstimate	Error %
0%	\$ 69.36	\$ 72.00	4%
2%	\$ 78.08	\$ 80.00	2%
4%	\$ 87.39	\$ 88.00	1%
6%	\$ 97.29	\$ 96.00	-1%
8%	\$ 107.72	\$ 104.00	-3%
10%	\$ 118.66	\$ 112.00	-6%

Table 3. Call Option Guesstimate and Exact Solution Comparison for different levels of δ

δ	Exact	Guesstimate	Error %
0%	\$ 99.25	\$ 96.00	-3%
2%	\$ 87.39	\$ 88.00	1%
3%	\$ 81.84	\$ 84.00	3%
6%	\$ 66.64	\$ 72.00	8%
8%	\$ 57.69	\$ 64.00	11%
10%	\$ 49.63	\$ 56.00	13%

A major weakness of the guesstimate is that it cannot be used to value options of differing moneyness or maturities. However, the guesstimate could be adjusted for time by multiplying r and δ by t and multiplying σ by the \sqrt{t} (Equation 5). Table 4 shows the guesstimates and errors for a 3-month call option with the same assumptions as the sample problem. The 3-month errors are generally less than 1%.

$$\tilde{C} = .4S(rt - \delta t + \sigma\sqrt{t}) \quad (5)$$

The guesstimate for the 3-month, at-the-money, call option is

$$\tilde{C} = .4 \times 1000(.04 \times .25 - .02 \times .25 + 0.20 \times .25^2) = \$42.00$$

Table 4. Call Option Guesstimate and Exact Solution Comparison for 3-month Maturity

σ	Exact	Guesstimate	Error %
10%	\$ 22.38	\$ 22.00	-2%
20%	\$ 42.14	\$ 42.00	0%
30%	\$ 61.90	\$ 62.00	0%
40%	\$ 81.66	\$ 82.00	0%
50%	\$ 101.42	\$ 102.00	0%
60%	\$ 121.18	\$ 122.00	0%
70%	\$ 140.94	\$ 142.00	0%
80%	\$ 160.70	\$ 162.00	0%
90%	\$ 180.46	\$ 182.00	0%
100%	\$ 200.22	\$ 202.00	0%

15-minute Classroom Exercise Plan using Call Option Guesstimates

The call option guesstimate can be used to introduce the Black-Scholes or the Binomial Option Pricing Models. Table 5 is a plan for the beginning of the class that introduces an option pricing model. The instructor begins by writing the guesstimate equation on the board and discussing its insight into risky-asset returns and call option valuation. The instructor then asks students to value several at-the-money options

without using a calculator. There are several option questions that that require guesstimation in Crack's Heard on *The Street: Quantitative Questions from Wall Street Job Interviews*. Crack suggests another estimation technique but it does not provide the intuition of the guesstimation equation. The drawbacks of the guesstimate should be highlighted to suggest that a more robust model is needed that does not make such simplistic assumptions about probabilities, discounting, and moneyness. Students should notice several similarities between the Black-Scholes and binomials equations and the guesstimate equation. Both the Black-Scholes formula and the Guesstimate assume stock returns have a risk-free return component ($r-\delta$) and a risky component that is a function of σ . The Black-Scholes and binomial models also incorporate probabilities using $N(d_1, d_2)$ and the risk-neutral probability of an up movement, respectively.

Table 5: 20-minute Lesson Plan for Call Option Guesstimate

Time	Activity
5 Minutes	Write guesstimate equations on board and discuss intuition
10 Minutes	Have students compute several guesstimates without a calculator
5 Minutes	Discuss guesstimate limitations, introduce Black-Scholes or binomial model, and compare formulas, assumptions, and estimates.

PUT OPTION GUESSTIMATION

Put options are contracts that give the owner the right, but not the obligation, to sell an asset at specified price and time. The guesstimate for at-the-money, one-year put options is

$$\tilde{P} = -.4S(r - \delta - \sigma) \quad (6)$$

The put option guesstimate can be interpreted as the present value of the puts option's expected payoff. The payoff of a put option is $K - S_T$ at maturity if the stock price is below the strike price or the payoff is zero if the price is above the strike. The term $S(r-\delta-\sigma)$ is the stock price drop if the stock price falls 1σ below its expected risk-free price appreciation ($r-\delta$). Like the call option guesstimate, the price change is multiplied by 0.40 to reflect the probability of a drop and the discount factor.

Put Option Sample Problem: What is the value of a one-year, at-the-money put option on the S&P 500 index? Assume that the S&P 500 index is \$1,000, the index dividend yield is 2%, the index has a standard deviation of 20%, and year one-year Treasury yields are 4%.

The guesstimate is

$$\tilde{P} = -.4 \times 1,000(.04 - .02 - .20) = \$72$$

The Black-Scholes put option formula (McDonald, 2006) is:

$$P = Ke^{-rt} N(-d_2) - Se^{-\alpha t} N(-d_1) \quad (7)$$

where d_1 and d_2 are given by Equations 3 and 4.

The Black-Scholes solution to the put sample problem is:

$$P = 1000e^{-0.04 \times 1} (0.50) - 1000e^{-0.02 \times 1} (.42074) = 67.99$$

where

$$d_1 = \frac{\ln(1,000/1,000) + \left(.04 - .02 + \frac{1}{2} .2^2 \right) 1}{.2\sqrt{1}} = .2 \text{ and } d_2 = .2 - .2\sqrt{1} = 0$$

$$N(-d_1) = .42074 \text{ and } N(-d_2) = 0.50$$

The put guesstimate tends to overvalue options compared to Black-Scholes. The guesstimation errors are 5% to 8% for a wide range of volatilities (Table 6), -1 to 11% for a wide range of risk-free rates, and -2 to 7% for a wide range dividend yields.

Table 6. Put Option Guesstimate and Exact Solution Comparison for different levels of σ

σ	Exact	Guesstimate	Error %
10%	\$ 29.77	\$ 32.00	8%
20%	\$ 67.99	\$ 72.00	6%
30%	\$ 106.27	\$ 112.00	5%
40%	\$ 144.33	\$ 152.00	5%
60%	\$ 219.28	\$ 232.00	6%
80%	\$ 292.06	\$ 312.00	7%

Table 7. Put Option Guesstimate and Exact Solution Comparison for different levels of r

r	Exact	Guesstimate	Error %
0%	\$ 89.16	\$ 88.00	-1%
2%	\$ 78.08	\$ 80.00	2%
4%	\$ 67.99	\$ 72.00	6%
6%	\$ 58.85	\$ 64.00	9%
8%	\$ 50.64	\$ 56.00	11%
10%	\$ 43.30	\$ 48.00	11%

Table 8. Put Option Guesstimate and Exact Solution Comparison for different levels of δ

δ	Exact	Guesstimate	Error %
0%	\$ 60.04	\$ 64.00	7%
2%	\$ 67.99	\$ 72.00	6%
3%	\$ 72.18	\$ 76.00	5%
6%	\$ 85.66	\$ 88.00	3%
8%	\$ 95.36	\$ 96.00	1%
10%	\$ 105.59	\$ 104.00	-2%

The put guesstimate could be adjusted for time by multiplying r and δ by t and multiplying σ by the \sqrt{t} (Equation 8). Table 4 shows the guesstimates and errors for the 3-month option. The 3-month errors are generally less than 1%.

$$\tilde{P} = -.4S(rt - \delta t - \sigma\sqrt{t}) \quad (8)$$

The guesstimate for the 3-month, at-the-money, put option is

$$\tilde{P} = .4 \times 1000(.04 \times .25 - .02 \times .25 - 0.20 \times .25^2) = \$38.00$$

Table 9 shows the guesstimate and exact Black-Scholes values for the sample put option of 3-month maturity. Errors are much smaller for this shorter term put option.

Table 9. Put Option Guesstimate and Exact Solution Comparison for 3-month Maturity

σ	Exact	Guesstimate	Error %
10%	\$ 17.41	\$ 18.00	3%
20%	\$ 37.15	\$ 38.00	2%
30%	\$ 56.89	\$ 58.00	2%
40%	\$ 76.60	\$ 78.00	2%
60%	\$ 115.88	\$ 118.00	2%
80%	\$ 154.87	\$ 158.00	2%

Option Guesstimate Alternative

An alternative specification for call and put option guesstimation is to omit the current asset value, S , from the guesstimate equation. The alternative sample call and put option guesstimates are then 8.8% and 7.2%. These can be interpreted as the price, as a percentage of the asset value, which can be applied to other options with similar volatility and dividend yield assumptions. It's useful to think about option price percentages when discussing hedging strategies. In the put example above, students can then see it costs about 7% to insure an index portfolio for one year.

MODIFIED DURATION GUESSTIMATION

Modified duration is a measure of the price sensitivity of a bond's price to yield movements. Longer term bonds are more sensitive to yield changes because discounting has a larger effect on value for more distant cashflows. Modified duration is calculated

by computing the weighted average timing of all the bond's payments. The modified duration of a zero-coupon bond is close to the maturity (in years) because all cashflows occur at maturity. The modified duration of a coupon bond is calculated by weighting each payment period (t) by the ratio of the present value of the each period's payment and the bond's total value. The modified duration of a coupon bond is less than the maturity date (T) and falls as the coupon rises.

The modified duration guesstimate is

$$\tilde{D} = (.8 - c)T \tag{9}$$

where c = annual coupon rate
 T = bond maturity in years

Modified Duration Sample Problem: What is the modified duration of a 10-year, 5% coupon bond that is selling for par?

The guesstimate of the 10-year bond is

$$\tilde{D} = (.8 - .05) \times 10 = 7.5 \tag{10}$$

The modified duration exact equation is

$$D = \left(\frac{1}{P_0(1 + YTM)} \right) \sum_{t=1}^T \left(\frac{C_t}{(1 + YTM)^t} \times t \right) \tag{11}$$

where

- P₀ = bond price
- T = maturity in years
- C_t = bond payment at time t (includes coupon and par payment)
- YTM = bond's yield to maturity

The modified duration of the 10-year, annual payment bond using Equation (11) and assuming a \$100 par amount is

$$D = \left(\frac{1}{100(1.05)} \right) \left(\frac{5}{(1.05)^1} \times 1 + \frac{5}{(1.05)^2} \times 2 + \frac{5}{(1.05)^3} \times 3 + \dots + \frac{5}{(1.05)^9} \times 9 + \frac{105}{(1.05)^{10}} \times 10 \right) = 7.7$$

Table 9 shows the duration guesstimates and exact solutions for 5% coupon bond with maturities of 1 to 30 years. The guesstimate underestimates short-term durations and overestimates longer-term durations. Table 10 shows the estimates and errors for a 10-year bond with coupon rates of 3 to 11%.

Table 9: Duration Guesstimate and Exact Solution Comparison for 5% Bond

T	Exact	Guesstimate	Error %
1	1.0	0.8	-21%
2	1.9	1.5	-19%
5	4.3	3.8	-13%
10	7.7	7.5	-3%
15	10.4	11.3	8%
20	12.5	15.0	20%
25	14.1	18.8	33%
30	15.4	22.5	46%

Table 10: Duration Guesstimate and Exact Solution Comparison by Yield/Coupon

T	C=YTM	Exact	Guesstimate	Error %
10	3.0%	8.5	7.7	-10%
10	5.0%	7.7	7.5	-3%
10	7.0%	7.0	7.3	4%
10	9.0%	6.4	7.1	11%
10	11.0%	5.9	6.9	17%

CONVEXITY GUESSTIMATION

Convexity is a measure of how a bond's duration changes as its yield to maturity changes. Positive convexity values imply that a bond's duration will decrease as its yield increases. Convexity increases exponentially for longer maturity bonds.

The convexity guesstimate is

$$\tilde{C} = \tilde{D}^2 \quad (12)$$

where \tilde{D} is the modified duration guesstimate of $(.8-c)T$

Convexity Sample Problem: What is the convexity of a 10-year, 5% coupon bond that is selling for par?

The guesstimate of the 10-year bond is

$$\tilde{C} = \tilde{D}^2 = 7.5^2 = 56.3 \quad (13)$$

The exact formula for convexity of an annual payment bond is

$$c = \frac{1}{P_0} \sum_{t=1}^T \frac{C_t t(1+t)}{(1+YTM)^{t+2}} \quad (14)$$

The convexity of the 10-year sample problem bond (assuming a \$100 par) is

$$c = \left(\frac{1}{100} \right) \left(\frac{5 \times 1 \times (1+1)}{(1.05)^{1+2}} + \frac{5 \times 2 \times (1+2)}{(1.05)^{2+2}} + \dots + \frac{5 \times 9 \times (1+9)}{(1.05)^{9+2}} + \frac{105 \times 10 \times (1+10)}{(1.05)^{10+2}} \right) = 75$$

Table 11 shows the convexity guesstimates and exact solutions for 5% bond maturities of 1 to 30 years. The guesstimate is most accurate for the 20-year bond, while the guesstimate underestimates short-term convexities and overestimates longer-term convexities.

Table 11: Convexity Guesstimate and Exact Solution Comparison for 5% Bond

T	Exact	Guesstimate	Error %
1	1.8	0.6	-69%
2	5.3	2.3	-57%
5	23.9	14.1	-41%
10	75.0	56.3	-25%
15	140.3	126.6	-10%
20	211.3	225.0	6%
25	282.5	351.6	24%
30	350.5	506.3	44%

SWAP GUESSTIMATES USING SIMPLE AVERAGES

Swaps are contracts that call for periodic payments based the difference between the actual price of an asset and an agreed upon swap price. A swap is similar to a series of forward contracts of varying maturity, however a swap uses a single swap price each period to determine payments. The calculation of a “fair” or “no arbitrage” swap price is tedious because it involves computing the weighted average of all forward prices where the weights are based on discount factors. The next several guesstimates of swap prices and swap rates are simple averages of forward prices.

Interest Rate Swap Guesstimate

Interest rate swaps exchange a periodic fixed payment for a floating rate payment. The fixed interest rate of an interest swap is commonly referred to as the swap rate. The guesstimate is the average of the spot and forward interest rates from the start of the swap to the maturity of the swap.

$$\tilde{R}_T = \text{Average}(\text{forward rates from period 1 to T}) \quad (15)$$

Interest Swap Rate Sample Problem: What is the 3-year no arbitrage swap rate if the 1, 2, and 3 year forward interest rates are 6, 7, and 8% respectively?

$$\tilde{R}_T = (6\% + 7\% + 8\%) / 3 = 7\%$$

The exact T-year swap rate equation is

$$R_T = \frac{\sum_{t=1}^T P_{0,t} r_{t-1,t}}{\sum_{t=1}^T P_{0,t}} \quad (16)$$

where T = Swap maturity (in year)

$P_{0,t}$ = Price of \$1 par, zero coupon bond of t-maturity (a.k.a., discount factor)

$R_{t-1,t}$ = forward interest rates on bonds issued at t-1 and mature at t

To compute the exact swap rate, you must first compute the zero-coupon bond prices for 1, 2, and 3 year maturities. The zero prices and the swap rate are

$$P_{0,1} = \left(\frac{1}{1.06} \right) = .943396$$

$$P_{0,2} = \left(\frac{1}{1.06 \times 1.07} \right) = .881679$$

$$P_{0,3} = \left(\frac{1}{1.06 \times 1.07 \times 1.08} \right) = .816369$$

$$R_3 = \frac{(.943396 \times .06 + .881679 \times .07 + .816369 \times .08)}{(.943396 + .881679 + .816369)} = 6.95\%$$

The exact solution of 6.95% is slightly less than the arithmetic average of 7.00%.

Currency Swap Guesstimate

Currency swaps require a periodic exchange of one currency for another. The currency swap exchange rate guesstimate is the average of the forward exchange rates.

The T-year swap exchange rate guesstimate is given by equation is

$$\tilde{R}_T = \text{Average}(\text{forward exchange rates from 1 to T}) \quad (17)$$

Currency Swap Rate Problem: Suppose you want to exchange U.S. dollars for euros in each of the next three years. What is the 3-year swap rate if the 1, 2, and 3 year forward exchange rates are \$1.50, \$1.60, and \$1.70 respectively?

$$\tilde{R}_T = (\$1.50 + \$1.60 + \$1.70) / 3 = \$1.60$$

The exact T-year currency swap rate is given by equation (28).

$$R_T = \frac{\sum_{t=1}^T P_{0,t} f_{0,t}}{\sum_{t=1}^T P_{0,t}} \quad (18)$$

where T = Swap maturity (in year)

$P_{0,t}$ = Price of \$1 par, zero coupon bond of t-maturity

$f_{0,t}$ = forward exchange rates

Using the interest rates from the previous sample problem, the currency swap rate is \$1.5952 as shown below. The exact solution is slightly less than the arithmetic average of \$1.60.

$$R_3 = \frac{(.943396 \times 1.50 + .881679 \times 1.60 + .816369 \times 1.70)}{(.943396 + .881679 + .816369)} = \$1.5952$$

Commodity Swap Price Problem

Commodity swaps make payments based on the difference between actual prices of specified assets and the agreed upon swap price. The guesstimate is

$$\tilde{R}_T = \text{Average}(\text{forward price from period 1 to T}) \quad (19)$$

Commodity Swap Problem: Suppose you want to exchange oil for a fixed dollar payment for the next three years. What is the 3-year oil swap rate if the 1, 2, and 3 year forward oil prices are \$126, \$130, and \$128 respectively?

The commodity swap guesstimate is

$$\tilde{R}_T = (\$126 + \$130 + \$128) / 3 = \$128$$

The exact T-year commodity swap price is again given by equation 18. Using the interest rates from the previous problems, the commodity swap price is \$127.95 as shown below. The exact solution is slightly less than the arithmetic average of \$128.

$$R_3 = \frac{(.943396 \times 126 + .881679 \times 130 + .816369 \times 128)}{(.943396 + .881679 + .816369)} = \$127.95$$

CURRENCY APPRECIATION GUESSTIMATES

The expected rate of currency appreciation of one currency in terms of another is determined by the relative inflation rates in the two countries. The currency of the

country with higher inflation is expected to depreciate and its currency will sell at forward discount to the spot exchange rate. The depreciating currency of the higher inflation country ensures that relative purchasing power is maintained and arbitrage eliminated.

The guesstimate is

$$\text{Appreciation(Depreciation) of Currency H in terms of Currency F} = (I_F - I_H) \quad (20)$$

where I_F = Expected inflation of country F
 I_H = Expected inflation of country H

Currency Appreciation Sample Problem: How much is the U.S. dollar expected to appreciate or depreciate relative to the euro if inflation levels in the U.S. and Germany are 3% and 5% respectively?

$$\text{Appreciation of U.S. dollars per euro exchange rate} = 5\% - 3\% = 2\%$$

The actual formula is derived from purchasing power parity that assumes exchange rates must adjust to maintain the price of the two goods in either currency.

The actual formula and solution to the sample is

$$\begin{aligned} \text{Appreciation of Currency H in terms of Currency F} &= (1+I_F)/(1+I_H)-1 & (21) \\ \text{Appreciation of U.S. dollars per euro exchange rate} &= 1.05/1.03-1=1.94\% \end{aligned}$$

The expected appreciation is also the forward premium according to the unbiased forward rate condition. The previous problem and guesstimate can also be applied to interest rate differentials. Table 12 shows the guesstimates and exact solutions for various levels of inflation.

Table 12: Convexity Guesstimate and Exact Solution Comparison for 5% Bond

I_H	I_F	Exact	Guesstimate	Error %
0%	50%	50%	50%	0%
2%	30%	27%	28%	2%
5%	20%	14%	15%	5%
10%	10%	0%	0%	0%
20%	5%	-13%	-15%	20%
30%	2%	-22%	-28%	30%
50%	0%	-33%	-50%	50%

THE USE OF GUESSTIMATES

There are several ways instructors can apply guesstimation. Guesstimates can be used to introduce complex topics, such as option pricing models, to provide insight into a

complex model. Students can use guesstimation as a test-taking strategy to create a range of feasible solutions or eliminate infeasible ones. Requiring students to guesstimate encourages critical thinking about the relationship between parameters and reasonable solutions. I often require students to make a quick guess about the solution of a problem before I allow them to write an equation or use a calculator. For example, I often ask students to give me a very quick guess of a bond's durations, a currency's forward premium, a call option's value, or swap's price. In these examples, I am determining whether students understand that durations cannot exceed the maturity or that high inflation currencies will sell at a discount in forward markets. Guesstimation is also useful technique in technical finance interviews where they are asked to estimate values without formulas or calculators. Lastly, guesstimation may be a more valuable career skill than the ability to make an exact calculation that requires little understanding.

ADVANCED GUESSTIMATION

Guesstimation can be enhanced by encouraging students to memorize key market values and benchmarks. Table 13 lists several key benchmarks and a reasonable range for their values.

Table 13: Key Finance Benchmarks

Benchmarks	Values
Market Risk Premium	3 - 8%
Large Cap Dividend Yield	1.5 - 2.0%
Large Cap Equity Annual Volatility	16 - 20%
1-Year Treasury Yield	Current Value
10-Year Treasury Yield	Current Value
S&P 500 Index	Current Value
Dow Jones Industrial Average	Current Value

SUMMARY

Guesstimates are quick and intuitive estimates to mathematical problems that can enhance student understanding of complex financial relationships. Students can use guesstimation immediately to improve performance on exams or in technical finance interviews. Guesstimation may lead to more lasting understanding and skill than simply solving complex formulas. The call and put guesstimates provide the intuition into the relationship between dividends, risk-free interest rates, volatility and option prices. The duration and convexity guesstimates show these values are related to maturity and coupon rate. The swap guesstimates provide the insight that swaps are substitutes for forward contract strips and swap prices are value-weighted averages of the forward prices. Lastly, the exchange rate guesstimate provides students the understanding that inflation with tend to devalue a currency and cause it to sell at a discount in forward markets. Guesstimation can be used in the finance curriculum to introduce new topics or to encourage critical thinking and judgment.

REFERENCES

Crack, T. F. *Heard on the Street: Quantitative Questions from Wall Street Interviews*, (2007), Timothy Falcon Crack

Hull, J. C., *Options, Futures and Other Derivatives*, (2008), 7th Edition, Pearson Education, Inc.

McDonald, R. L. *Derivatives Markets*, (2006), 2nd Edition, Pearson Education, Inc.