IMPLIED RISK ADJUSTED DISCOUNT RATES

Patricia A. Ryan, Colorado State University
Timothy J. Gallagher, Colorado State University

Abstract

Investors have become increasingly concerned about their investment portfolios. This article shows how the certainty equivalent methodology can and should be properly used to arrive at an implied risk adjusted discount rate. The methodology separates the time and risk elements, the former being compounded by the latter. The implied risk adjusted discount rate can rise rapidly as the certainly equivalent factors decreases. This indicates that it is likely that risk adjusted discount rates that use a simple adjustment factor do not properly reflect the appropriate risk and in many cases may underestimate an investment's true risk.

INTRODUCTION

Ease of application does not justify model choice the goal of the investor is to maximize personal wealth. Although there has been debate in the literature about how to appropriately adjust for risk in the cost of capital, few disagree that certainty equivalents (CE) are the theoretically better than risk adjusted discount rates (RADR) for discounting risky cash flows; however, most choose to apply the simpler RADR and sidestep the importance of the CE methodology. Scott, Martin, Petty, and Keown (1999) comment the reason that RADR is more popular than certainty equivalent risk adjustment is "purely and simply its ease of implementation". If the theoretically superior certainty equivalent model can be used to develop a simple implied risk adjusted discount rate, the implied rate would be superior to a simple RADR, which combines the elements of time and risk. Such an implied rate could vary across time and according to projected certainty equivalents and provide a superior capital budgeting tool. The CE methodology is superior in defining and measuring these risks. The goal of this article is to review the basis of each methodology and develop an implied risk adjusted certainty equivalent discount rate aimed to assist the manager make capital investment decisions.[1]

Everett and Schwab (1979) allege the use of RADR is a 'gross oversimplification'. Robichek and Myers (2001) discuss the time and risk elements of the RADR methodology. In alignment with this commentary, it is clear the time and risk elements must be separated in order to properly evaluate a capital budgeting project for N periods.

As Damodaran (2005) states, the cost of capital is typically the only input valuation in which we adjust for risk. He argues that since this is the only input variable sensitive to risk and the market risk premium reflects nondiversified risk, the payoff for risk management is hard to trace. It is arguably important to separate the risk and time components.

Many finance textbooks place greater emphasis on risk-adjusted discount rates than the certainty equivalence methodology. Correspondingly, text problems emphasize the use and application of RADR over certainty equivalence. It is postulated that certainty equivalents are superior to RADR and easily applicable in personal financial planning. When there are instances of unusually high discount rates and multiple time period models, CE provides different economic decisions to RADR.

The RADR method simultaneously adjusts for time and risk whereas the certainty equivalent method separates the two (Beedles and Joy, 1978). When there is a simultaneous adjustment for time and risk, there is an interplay between the two that is not consistent with economics principles. For example, if the RADR is assumed to be 10% of which 4% is the risk free rate and 6% is the adjustment for risk. The risk adjustment has a largely different effect depending on the number of periods of investment. In large, multi-year capital investment projects, the risk adjustment using the RADR is arguably not an accurate representation of the risk adjustment. The certainty equivalent method provides one means to fix this problem be extricating time and risk into two separate components.

Gitman (1995), Sick (1986), and Megginson (1997) agree CE is theoretically superior to RADR. Both note the popularity of RADR stems from two main issues: acceptance by financial decision makers and ease of estimation and application. Firms like to develop several risk classes and then force all projects into one of these classes. Is this terrible wrong? Not if errors cancel out across projects and estimation bias is limited. This is generally not the case, however. Many texts present both certainty equivalence and RADR and then comment that RADR is preferred in practice. Ease of use may not be worth the sacrifice of accurate assessment of risk. Financial planners should be aware of the large changes in the implied interest rate that result from significant decreases in the certainty equivalent, especially over shorter investment horizons.

This article illustrates how certainty equivalence should be preferred over risk-adjusted discount rates in cost of capital calculations. The article proceeds as follows. Section II the background of certainty equivalence and risk adjusted discount rates, Section III presents illustrative examples and theoretical development, Section IV develops the implied risk adjusted rate that allows for different certainty equivalent proportions and risk free discounting for capital investment decisions, and Section V concludes.

CERTAINTY EQUIVALENCE AND RISK ADJUSTED DISCOUNT RATES

In examining a capital budgeting decision using certainty equivalents, the cash flow (in the numerator of the present value calculation) is adjusted to reflect the risk of the cash flow. Once this risk adjustment is made, the cash flow is discounted at the risk free rate to reflect time differentials. This methodology appropriately separates the time and risk factors, allowing for linear adjustments for risk. The certainty equivalent is the value of a certain prospect that yields the same level of utility as the expected utility of an uncertain prospect. For the risk averse investor, this value will always be lower than the expected value of a risky positive cash flow. On the other hand, RADR methodology does not adjust the cash flow in the numerator, but rather adjusts the discount rate in the denominator. The implicit assumption in RADR is that risk increases as time increases as developed in Harris and Pringle (1985) and others.

The debate about appropriate risk adjustment is not new in the finance literature. Robichek and Myers (1966) discussed the problems associated with RADR and since that time, there have been numerous studies published that address the difficulties of application of RADR. Fama (1977) discussed the valuation of multi-period cash flows. Lewellen (1977, 1979) argued that risky outflows require higher RADR's, while Celec and Pettway (1979) and Hartl (1990) argue the opposite. Berry and Dyson (1980, 1983) and Booth (1982, 1983) continue this debate. Beedles (1978 a, b) suggests that certainty equivalents are superior for estimating the present value of risky cash outflows and Miles and Choi (1979) debate his conclusion. This paper does not center on this debate; rather we present an application of certainty equivalents that appropriately adjust for risk and provide an implied risk adjusted discount rate. Gallagher and Zumwalt (1991) illustrate how large negative discount rates applied to risky cash outflows may lead an unbounded present value and extreme sensitivity to the number of time periods. Most recently, Ariel (1998) argues the application of very low discount rates are for risky cash outflows is inconsistent with market completeness and the basic tenets of the CAPM. This research is not about this debate directly, but rather, we develop an implied risk adjusted discount rate applying certainty equivalent factors and the risk free rate of interest, which separates the dimensions of time and risk. While concerns with RADR are prevalent in multi-period models, Hull (1986) showed the single period model is not exempt from problems because the RADR and the risk characteristics that correspond to risky cash flows may differ.

Implications of RADR application are numerous. First, there is not a clear sign in the literature as to the direction of adjustment for risky cash outflows. While there is a general consensus that an upward adjustment is most appropriate for risky cash inflows, there is not general agreement in the literature as to handle risky cash outflows. Most corporate finance texts advocate lower RADR's for risky cash outflows. Second, the application of a risk adjustment with RADR's is highly arbitrary; most texts advocate a +/- 2 percentage point risk adjustment. We will show this adjustment is not nearly enough if the certainly equivalent is below 0.95. We advocate that managers apply certainty equivalents in order to more fully grasp the true risk of an investment and to adequately separate the risk and timing components. Such an application could have limited exposure to recent turmoil in the technology industry; an industry with arguably low certainty equivalents.

Utility functions are not an issue with certainty equivalents or RADR. The necessary variables include the end of period cash payoff, the quantifiable amount of risk, the risk free rate of interest and the price of risk as determined by the market. Since classes of individuals comprise the market, the composite of those classes can quantify individual risk classes. Therefore, the separation theorem allows for separation of calculation from attitudes toward risk. Interestingly, if there were to exist an efficient secondary market, only one-period analysis would be necessary. However, this is not generally the case, especially with projects that involve large negative cash outflows many years in the future.

The certainty equivalent level of wealth is the amount at which the investor is indifferent between the risky outcome and the risk free outcome. The investor decides what risk free cash flow he would be willing to accept in exchange for a risky cash flow. For example, if an investor has a 1/300 probability of winning a $10 million lottery, the certain equivalent for a risk neutral investor would be $33,333.33, which is the expected value of a fair game. The period before the drawing of the lottery winner, one has the choice to cash out. What amount would he require to cash out and leave the game? The cash out amount is the certainty equivalent. Given a risk averse utility curve, the certainty equivalent might be $20,000 or some similar number, which is considerably below the expected value of $33,333.33. The certainty equivalent of $20,000 is the cash flow of $33,333.33 adjusted for risk. This amount is then adjusted for time value, by discounting at the risk free rate.

A possible complication of CE implementation is discussed in Brigham, Gapinski, and Daves (1999). They state there is "no practical way to estimate certainty equivalents". Perhaps it should be recognized that adjusting the weighted average cost of capital by a risk factor, while easy, can hide the investor's need to assess the risk of the project at hand and is also subjective. We argue that ease of implementation on the part of the investor or the financial planner should not cause the sacrifice of accurate implied discount rates. The problem is most severe with large interest rates and with low certainty equivalents.

CERTAINTY EQUIVALENCE AS A SUPERIOR RISK ADJUSTMENT TOOL

Brealey and Myers (1991) state the use of a single risk-adjusted discount rate for long-lived assets will not work if there are multiple phases of project design, in essence presenting a binomial model. If market risk were to change over the life of the project, RADR will not accurately depict the new level of risk.

Additionally, to use the same RADR for distant cash flows implies they are higher risk, which is contrary to the truth for many projects. The discount rate should compensate for the risk accepted per period, the more distant the cash flows, the more periods, and the larger the total risk adjustment using RADR. Further, Ben-Horim and Sivakumar (1988) show RADR leads to biased NPV calculations. The same RADR should only be used if a project has the same market risk at each point in its life. Often market risk changes as real options are exercised, and RADR improperly measures true risk on a period by period basis.

It is recognized that certainty equivalents are accepted, at least in theory, as a superior risk adjustment model to risk adjusted discount rates. Their limited usage in both personal and corporate financial management is blamed on the relative difficulty of application and determination of certainty equivalents for risky cash flows. In the next section, we will show how the certainty equivalent methodology can and should be properly used to arrive at an implied risk adjusted discount rate for the investor. This methodology separates the time and risk elements, which are compounded in the risk adjusted discount rate methodology. Kudla (1980) showed that certainty equivalent factors cannot be assigned solely based on the variance of the cash flows, but rather should be based on systematic risk. Investors and financial planners need to develop tools to adequately estimate the certainty equivalent of risky capital budgeting projects. The implications affect both investors and corporations and are especially important to the investor as they try to quantify the risk characteristics associated with capital budgeting projects.

AN IMPLIED RISK ADJUSTED RATE

Haley (1984) and others show the RADR model implicitly assumes the risk of the cash flows increase with time. Thus, we develop an implied risk adjusted rate that separates time and risk. Figures 1-3 illustrate various certainty equivalents and implied interest rates for periodic model, specifically for the risk free rates of 5%, 8%, and 10%, respectively. The model assumes an expected but uncertain cash flow of $1,000 with CE factors (α) ranging from .95 to .05. The CE is calculated by multiplying the cash flow by the CE factor (α). The present value of the certainty equivalent (PV CE) is calculated by dividing the CE by $(1+R_f)$. Thus:

$$NPV = \sum_{i=1}^{n} \frac{CECF}{(1+Rf)^n} \text{ - Initial Outlay} \tag{1}$$

The implied rate is calculated as follows: $\{[(1+R_f)/\alpha] - 1\}$. For initial purposes, a check is built into the model and is calculated as follows: (CE/PV CE). The annual rate for the one period model is the previously calculated implied rate. For other models, the annual rate is $\{[(1+\text{implied rate})^{1/N}] - 1\}$. The differences generally occur in multiple period models.[2] Likewise, the risk adjusted discount rate implies the following form of discounting:

$$NPV = \sum_{i=1}^{n} \frac{CF}{(1+RADR)^n} \text{ - Initial Outlay} \tag{2}$$

Since the cash flow is discounted by the RADR, time and risk are not separated and thus may not compound properly. The CE method allows for the separation of risk and time by placing risk in the numerator and time in the denominator and discounting at the risk free rate.

[Insert Figures 1-3 about here]

Figure 1 shows a graphical representation of the implied risk adjusted rate using 1, 2, 3, 10, and 20 periods for the 5% risk free rate. As is seen, the implied rate rises nearly exponentially as the number of period increase and the CE factor decrease. The greater the risk aversion, the lower the certainty equivalent. For illustrative purposes, we have included one set of numerical calculations in Table 1. In these calculations, we show the 1 period model for a risk free rate of 5% with certainty equivalents ranging from 0.95 to 0.05. The implied rate for a 0.95 CE is 10.53%, representing a 5% reduction for risk and a 5% discount rate. The table shows a dramatic increase in implied rates as the CE's decrease. This increase in implied rates is perhaps greater than intuitively expected. It is definitely larger than the common +/- 2 percentage point fudge factor used in RADR which illustrates why RADR is not an accurate model for many investment decisions. For example, a CE of 0.80 and a risk free rate of 5% result in an implied rate of 31.25%. With a CE of 0.50, the implied rate soars to 110%. This is neither complex nor difficult, but illustrative of how the implied rate increases significantly with decreases in CE's. Intuitively pleasing, this also allows for the separation of risk and time. It is also unlikely that RADR's are adjusted in this manner to reflect the risky cash flow of a period. Figures 2 and 3 further illustrate how the implied rate is an increasing function of the risk free rate for risk free rates of 8% and 10%, respectively. Again, we see implied rates rise exponentially as CE factors decrease and secondly as the number of periods increase. The graphs illustrate that implied interest rates very sensitive to the certainly equivalent, even for mid term investments of 2, 3, and 10 years. Since investors are generally interested in returns over more than one year, multi-period models are important.

[Insert Table 1 about here]

CONCLUSION

Certainty equivalents are a superior risk adjustment model over risk-adjusted discount rates because certainty equivalents allow for the separation of time and risk. Their limited usage in investment management is blamed on the relative difficulty of application and determination of certainty equivalents for risky cash flows. We show why the certainty equivalent methodology can and should be properly used to arrive at an implied risk adjusted discount rate. This methodology separates the time and risk elements, which are compounded in the risk adjusted discount rate methodology. Kudla (1980) showed that certainty equivalent factors cannot be assigned solely based on the variance of the cash flows, but rather should be based on systematic risk. We depict the need for financial managers to develop tools to adequately estimate the certainty equivalent of risky capital budgeting projects. Furthermore, we see how rapidly the implied risk adjusted discount rate rises as the certainly equivalent decreases. The results of this paper indicate it is likely that risk adjusted discount rates that use a simple adjustment factor do not properly reflect the appropriate risk and in many cases may underestimate corporate risk, while compounding excessive some periods of risky cash flows. Such implied discount rates are superior to RADR, both in theory and application. Usage of such methodology might allow investors to better recognize the risks inherent in certain investments and allocate their funds accordingly. The application is especially important to the investor as they try to quantify the risk characteristics associated with investment decisions.

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Figure 1: Implied risk adjusted rates using risk-free rate of 5% for 1, 2, 3, 10 and 20 years.

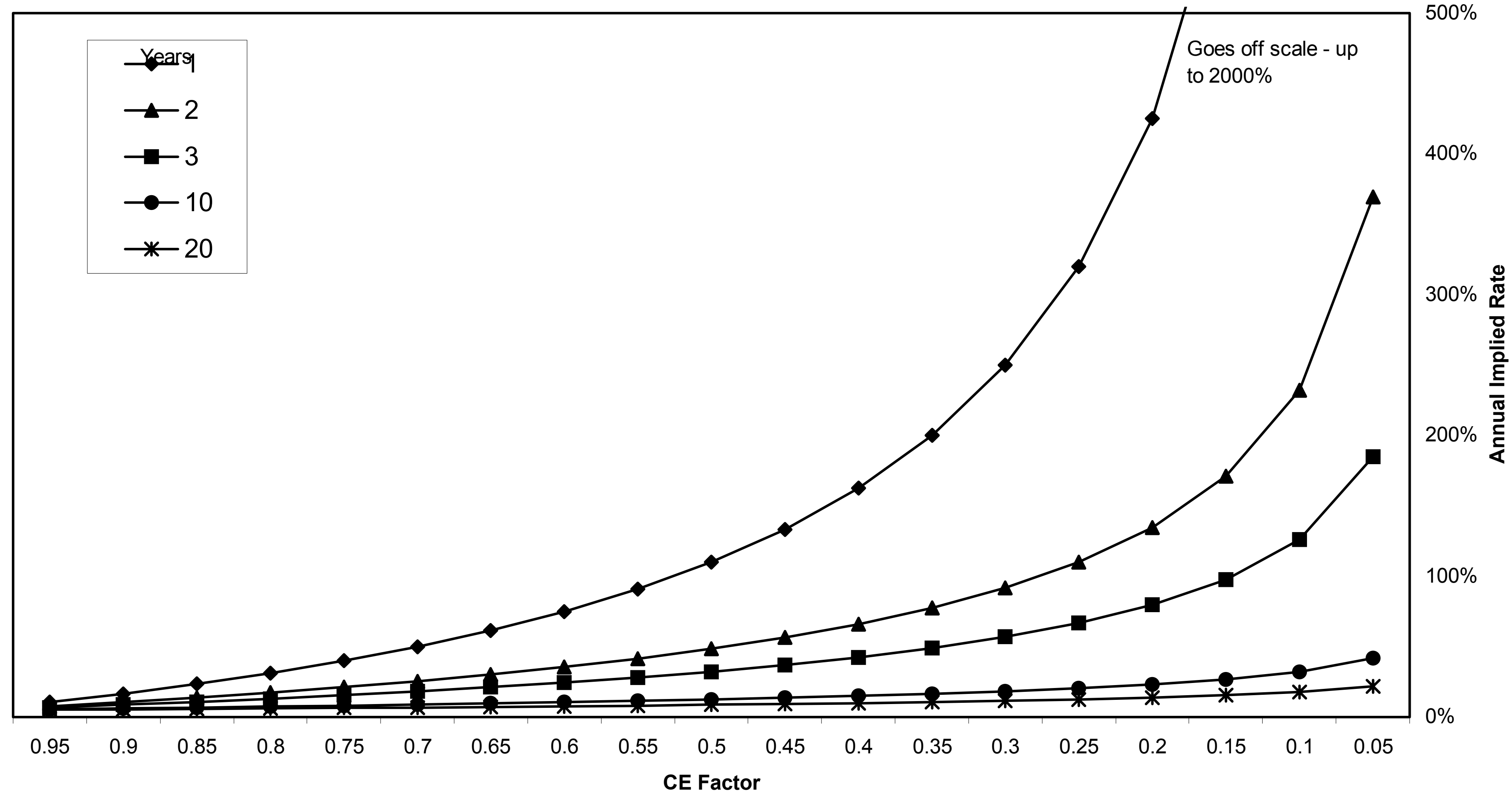

Figure 2: Implied risk adjusted rates using risk-free rate of 8% for 1, 2, 3, 10, and 20 years.

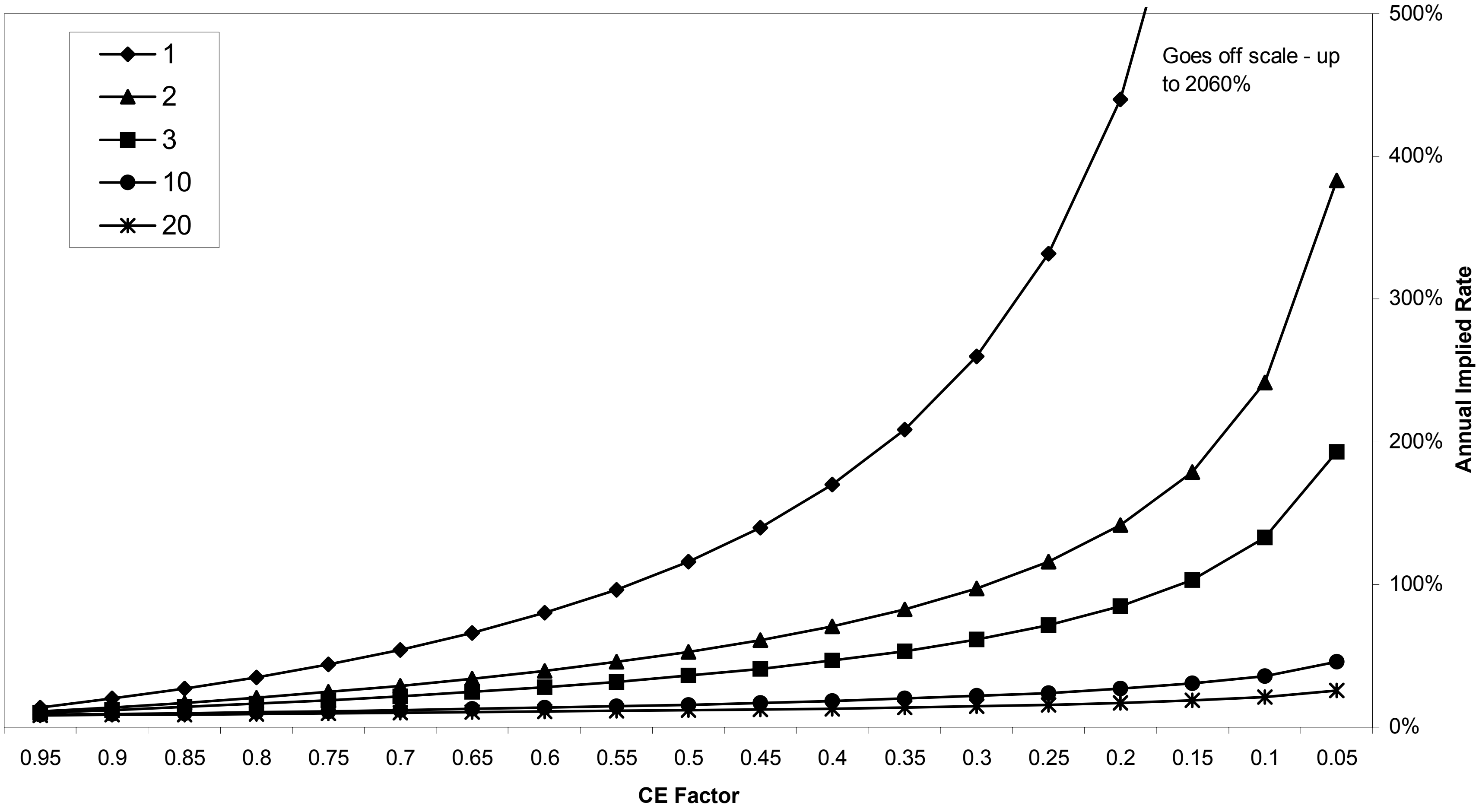

Figure 3: Implied risk adjusted rates using risk-free rates of 10% for 1, 2, 3, 10, and 20 years.

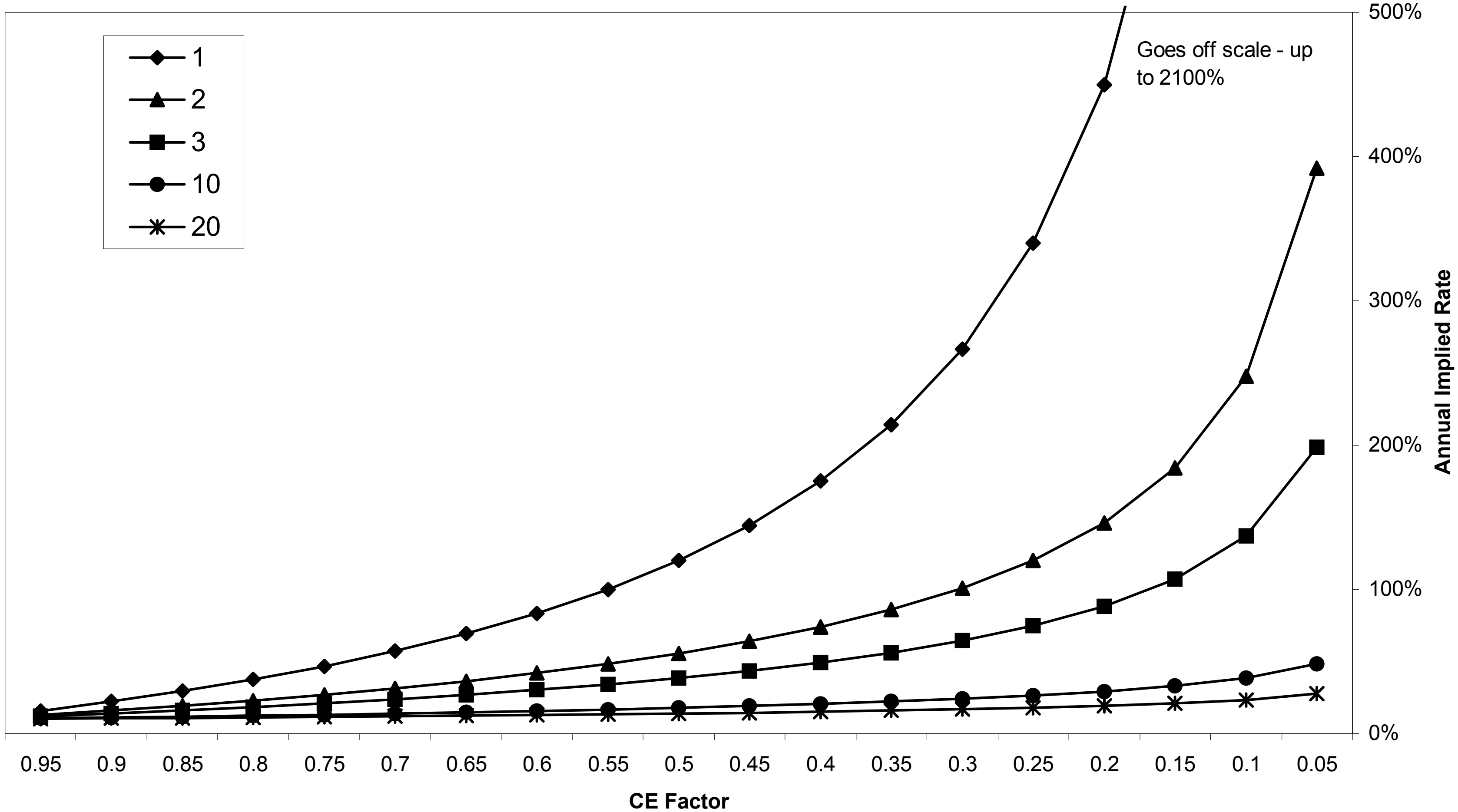

Table 1: Implied risk adjusted discount rates using a risk free rate of 5% illustrated for 1 period. The model assumes a cash flow of $1,000 with CE factors (α) ranging from .95 to .05. The CE is calculated by multiplying the cash flow by the CE factor (α). The present value of the CE (PV CE) is calculated by dividing the CE by $(1+R_f)$. The implied rate is calculated as follows: $\{[(1+R_f)/\alpha] - 1\}$. The check is calculated as follows: (CE/PV CE). The annuity rate for the 1 period model is the previously calculated implied rate. For other models, the annuity. rate is $\{[(1+\text{implied rate})^{1/N}] - 1\}$.

Cash Flow	CE-factor	CE	Risk-free	PV CE	Implied Rate	Check	Ann Rate
$1,000	0.95	$950	0.05	$905	10.53%	10.53%	10.53%
$1,000	0.9	$900	0.05	$857	16.67%	16.67%	16.67%
$1,000	0.85	$850	0.05	$810	23.53%	23.53%	23.53%
$1,000	0.8	$800	0.05	$762	31.25%	31.25%	31.25%
$1,000	0.75	$750	0.05	$714	40.00%	40.00%	40.00%
$1,000	0.7	$700	0.05	$667	50.00%	50.00%	50.00%
$1,000	0.65	$650	0.05	$619	61.54%	61.54%	61.54%
$1,000	0.6	$600	0.05	$571	75.00%	75.00%	75.00%
$1,000	0.55	$550	0.05	$524	90.91%	90.91%	90.91%
$1,000	0.5	$500	0.05	$476	110.00%	110.00%	110.00%
$1,000	0.45	$450	0.05	$429	133.33%	133.33%	133.33%
$1,000	0.4	$400	0.05	$381	162.50%	162.50%	162.50%
$1,000	0.35	$350	0.05	$333	200.00%	200.00%	200.00%
$1,000	0.3	$300	0.05	$286	250.00%	250.00%	250.00%
$1,000	0.25	$250	0.05	$238	320.00%	320.00%	320.00%
$1,000	0.2	$200	0.05	$190	425.00%	425.00%	425.00%
$1,000	0.15	$150	0.05	$143	600.00%	600.00%	600.00%
$1,000	0.1	$100	0.05	$95	950.00%	950.00%	950.00%
$1,000	0.05	$50	0.05	$48	2000.00%	2000.00%	2000.00%

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[1] The application in corporate finance is equally clear when the financial manager needs to examine several mutually exclusive projects within a budget constraint.
[2] See Copeland and Weston (1988) for a detailed derivation.