GRAPHIC REPRESENTATION OF OPTION PORTFOLIOS BY CONNECTING THE DOTS

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Abstract

This paper develops a simple method for constructing payoff diagrams for option portfolios. The method is based upon the fact that the payoff diagram is a piecewise linear function. This method is easier to use and understand than the graphical or algebraic methods described in most investment textbooks.

INTRODUCTION

In most investment textbooks (see Hull, 2003; Levy, 1996; Sharpe, Alexander, and Bailey 1995; Smith, Proffit, and Stephens 1992) option strategies are analyzed and compared by determining the portfolio payoff at expiration as a function of the price of the underlying stock. The graph of the portfolio payoff versus the price at expiration is called the payoff diagram. From the payoff diagram the profit diagram can easily be determined by subtracting the price of the portfolio and other costs, like taxes, commissions, and interest expense. The payoff diagram is useful for analyzing the risk-return characteristics of option portfolios (Gastineau, 1979), and designing strategies that are tailored to an investor□s risk preferences and stock market performance expectations (de Marco, 1995).

There are two approaches for determining the payoff diagram: the algebraic and graphical methods. For most students, the algebraic method is difficult because it requires the ability to add together piecewise continuous functions and then to plot the resulting function. Although the graphical method is easier and leads directly to the payoff diagram, this method can be tedious and prone to error for complicated positions, because of the large number of figures that must be plotted on the same graph and then added together. Also, it has been my experience that many students find it difficult to add figures together.

This paper develops a simple method for constructing payoff diagrams for option portfolios. This method is based upon the fact that the payoff diagram is a piecewise linear function. This method is easier to use and understand than the graphical or algebraic methods, described in most investment textbooks.

ALGEBRAIC AND GRAPHICAL METHODS

The algebraic method involves three steps: 1) identify a set of critical price ranges over which the payout diagram is linear, 2) determine the algebraic representation of the payout function for each position at each critical price range, and 3) sum over all positions the payout functions at each critical price range. The end result is a piecewise linear function. For example, consider the *Butterfly Spread* example from Gastineau (1979, pages 115-123), shown below:

	Price Range			
Position	$P \le 35$	$35 < P \le 40$	$40 < P \le 45$	$P > 45$
Buy 1 Jan 35 call at 8	0	P – 35	P - 35	P - 35
Sell 2 Jan 40 calls at 4 □	0	0	-2(P - 40)	-2(P - 40)
Buy 1 Jan 45 call at 1	0	0	0	P - 45
Portfolio	0	P – 35	-P + 45	0

The graphical method requires that the student draw the payout diagram for each position and then add the diagrams together. A position is the purchase or sale of a number of units of a stock, risk-free bond, call, or put option. For the Butterfly Spread, the graphical method is illustrated in Figure 1. According to Gastineau (1979) the graphical method is superior, because algebraic formulations are difficult to understand, prone to error, and difficult to interpret. However, for complicated positions the graphical method is tedious and also prone to error, because of the large number of figures that must be plotted on the same graph and then added together.

[Insert Figure 1 here]

CONNECTING THE DOTS

Our method is a combination of the graphical and algebraic methods. First, the portfolio payoff at the expiration date is evaluated at a set of critical prices, consisting of the exercise price of each option, a price lower than the lowest exercise price, and a price greater than the greatest exercise price. The payoff diagram is simply the line connecting the payouts at the critical prices

It is easy to see that this approach yields the same payout diagram as either the algebraic or graphical methods. Portfolios are formed by buying and selling combinations of the stock, risk-free asset, calls, and puts. Between any two adjacent exercise prices, the portfolio payout function is linear. This is true because the payout for any position between exercise prices is also linear, and the portfolio payout is a linear combination of its component payouts. Because two points determine a line, the portfolio payout between, and including, any two adjacent exercise prices is the straight line connecting the portfolio payout at each exercise point. Therefore, the critical points consist of the set of exercise prices, a price lower than the lowest exercise price, and a price greater than the greatest.

This approach is superior to the algebraic method because it is much easier to determine the value at a given price than to determine an algebraic equation describing the payout over some range. Further, the difficult problem of determining the critical price ranges is eliminated. This approach is superior to the graphical method because it eliminates the need to plot a graph for each security and eliminates the task of adding figures together, which some students find difficult. Obviously, it is much easier to add values than either figures or algebraic expressions.

EXAMPLES

The following examples illustrate the simplicity of the approach. The *Butterfly Spread* example is taken from Gastineau (1979, pages 115-123). The *Put-Call Parity* example shows how a risk-free bond can be constructed by writing a covered call and buying a put with the same exercise price. The *Big W* example illustrates the ease by which the payoff diagram for a complicated portfolio can be determined.

1. *The Butterfly Spread*

	Payoff at Critical Prices				
Position	0	35	40	45	50
Buy 1 Jan 35 call at 8	0	0	5	10	15
Sell 2 Jan 40 calls at 4 ▯	0	0	0	-10	-20
Buy 1 Jan 45 call at 1	0	0	0	0	5
Portfolio	0	0	5	0	0

2. *Put-Call Parity*

	Payoff at Critical Prices				
Position	0	35	40	45	50
Buy 1 share of stock	0	35	40	45	50
Sell 1 Jan 40 call	0	0	0	-5	-10
Buy 1 Jan 40 put	40	5	0	0	5
Portfolio	40	40	40	40	40

3. ***The Big W***

Position	Payoff at Critical Prices						
	0	5	10	15	20	25	30
Buy $10 zero-coupon	10	10	10	10	10	10	10
Sell 2 Jan 5 calls	0	0	-10	-20	-30	-40	-50
Buy 3 Jan 10 calls	0	0	0	15	30	45	60
Sell 2 Jan 15 calls	0	0	0	0	-10	-20	-30
Buy 3 Jan 20 calls	0	0	0	0	0	15	30
Sell 2 Jan 25 calls	0	0	0	0	0	0	-10
Portfolio	10	10	0	5	0	10	10

The payoff diagram for the *Big W* strategy, shown in Figure 2, is easily drawn by connecting the points determined by the portfolio payoff at the critical prices. In contrast, the graphical method, shown in Figure 3, involves the addition of six payoff diagrams. Likewise, the algebraic method, demonstrated in Table 1, is at least as cumbersome because it requires that students add together six piecewise continuous functions, defined over six price segments.

[Insert Figures 2 and 3 here]
[Insert Table 1 here]

CONCLUSION AND EXTENSION

This paper describes a simple method for constructing payoff diagrams for option portfolios. As we have shown, this method is much easier to understand and use than either the graphical or algebraic methods used in investment and derivatives textbooks. This method can also be used to design an option portfolio that has a specified payout diagram. For a given feasible payout diagram, we know the payout at each critical point. Then by solving a system of equations we can determine a portfolio (nonunique) that generates the desired payout at each critical point. This approach was used to generate the *Big W* portfolio.

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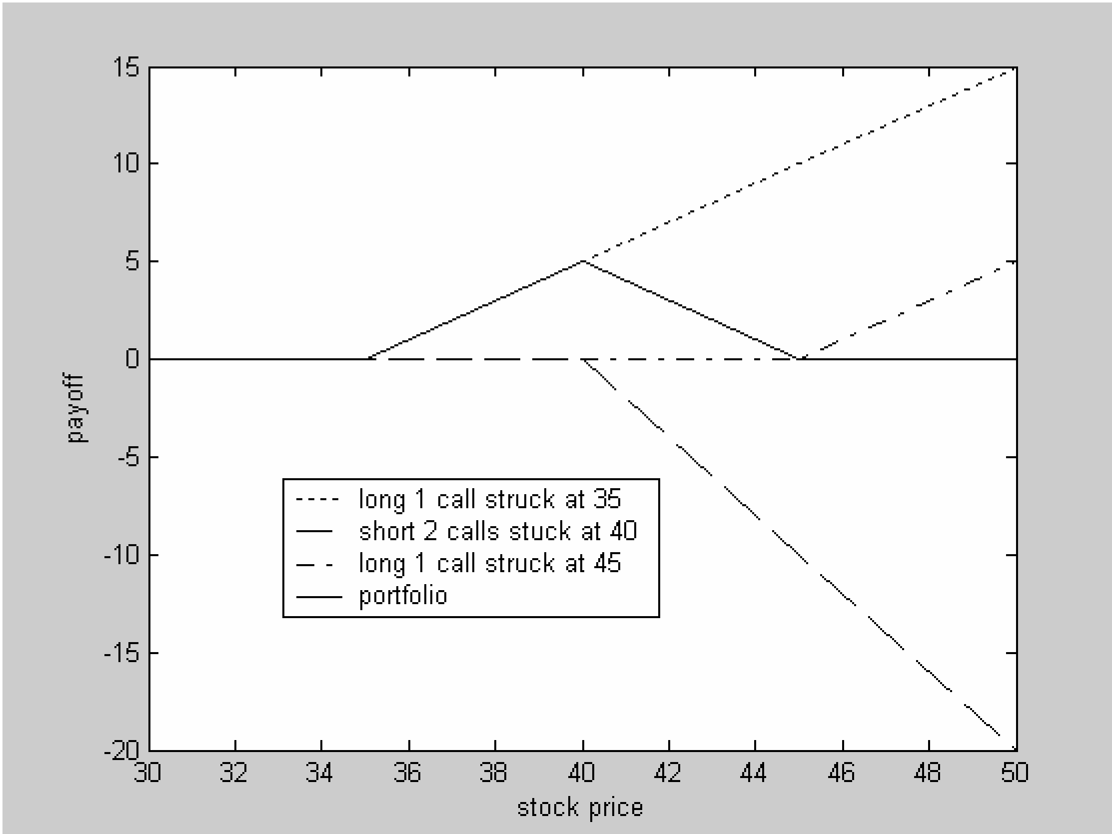

Figure 1 Graphical Construction of Butterfly Spread

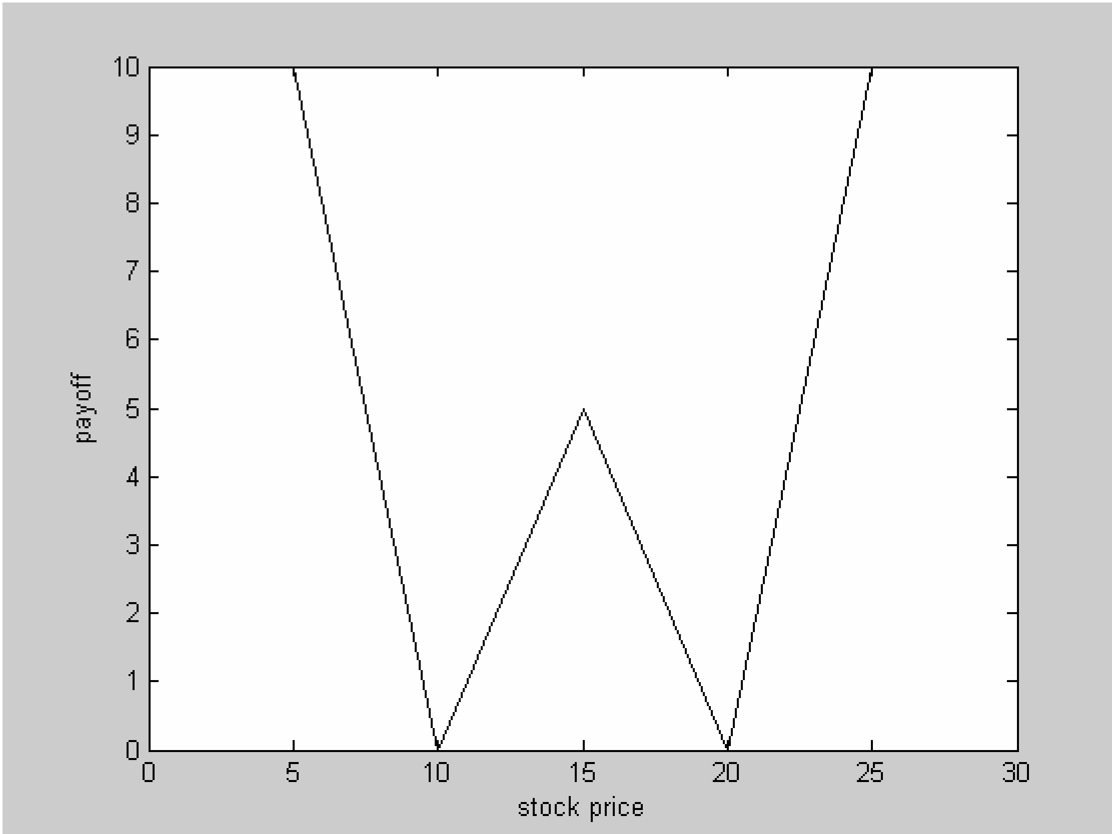

Figure 2 Payoff for Big W

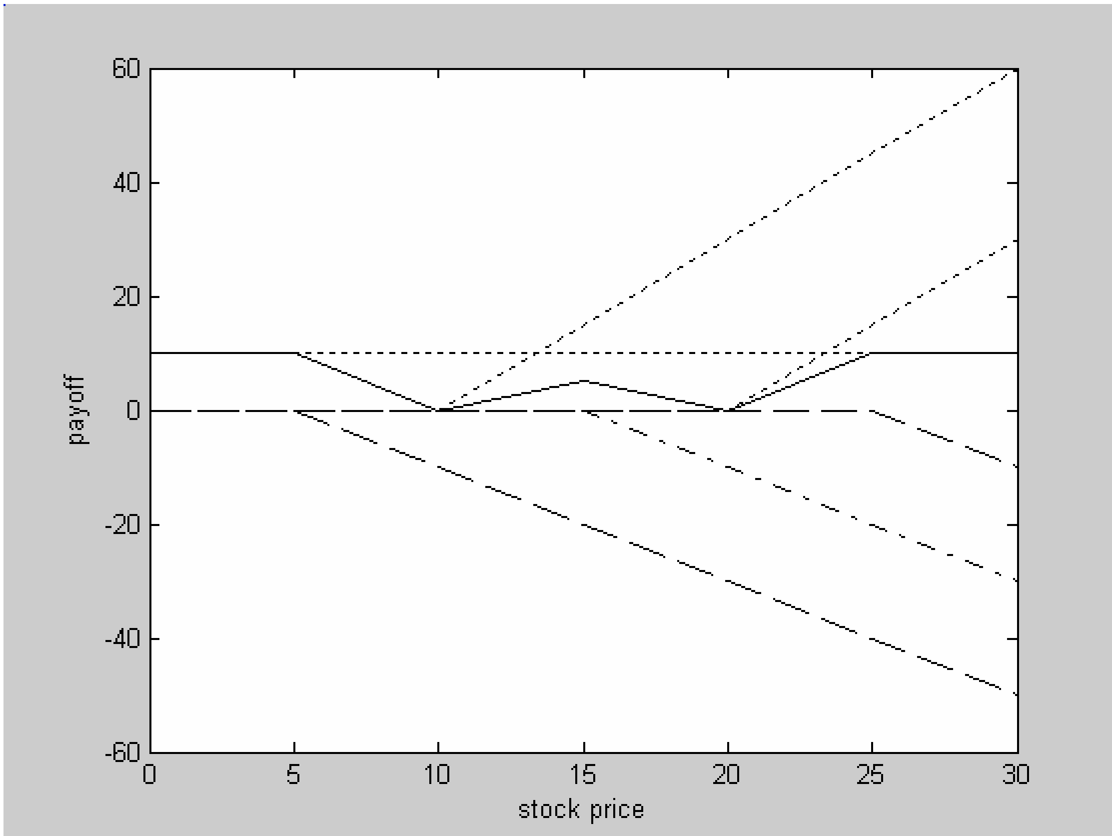

Figure 3   Graphical Construction of Big W

Table 1 Payoff Table for Big W

	Price Range					
Position	P ≤ 5	5 < P ≤ 10	10 < P ≤ 15	15 < P ≤ 20	20 < P ≤ 25	25 < P
Buy $10 zero-coupon	10	10	10	10	10	10
Sell 2 Jan 5 calls	0	-2(P – 5)	-2(P – 5)	-2(P -5)	-2(P – 5)	-2(P – 5)
Buy 3 Jan 10 calls	0	0	3(P – 10)	3(P – 10)	3(P – 10)	3(P – 10)
Sell 2 Jan 15 calls	0	0	0	-2(P – 15)	-2(P – 15)	-2(P – 15)
Buy 3 Jan 20 calls	0	0	0	0	3(P – 20)	3(P – 20)
Sell 2 Jan 25 calls	0	0	0	0	0	-2(P - 25)
Portfolio*	10	20 – 2P	-10 + P	20 - P	-40 + 2P	10

*The portfolio payoff is the sum of the six positions