$\frac{ڑ}{ٻٻ}$ ٻٺ

Students sometimes may include C within the brackets, multiplying 1 and i/12, before powering, or sometimes they only power i/12.

Equations of annuities likeὼ $\frac{ط}{ٻ،ٿٿ}$ 2500, where x is the payment cannot be solved, because the structure of the expression on the left side of the equations is not clear.

Another difficulty might be the gaining intuition of compound interest formula, which involves factoring an expression like:

C + C r = C (1 + r), and

C (1 + r) + C (1 + r) r = C (1 + r) (1 + r) = C (1 + r $)^2$; and so on.

Some of the freshmen do not have the sufficient structure sense to recognize a simple structure and substitute (1 + r) as a single entry.

Two research questions guided the present study: 1) Do freshmen have structure sense?, and 2) How will the lack, if any, of structure sense be related to their difficulties in algebraic manipulations and courses like finance, administration and operations?

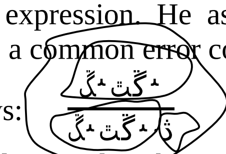
A diagnostic mathematics test, to 270 freshmen, in a private university in Mexico, showed a low percentage of correct answers. Many of freshmen's errors were due to a low understanding of structure sense. This study highlights the most worrying structures sense lacks for those students. This paper does not pretend to be exhaustive; it will analyze some freshmen's work in simplifying and manipulating algebraic expressions and solving equations, which clearly manifest a poor structure sense and which represent key problems. A further study was made looking directly on structure sense in several algebraic expressions.

First I provide an overview of definition and errors of structures sense reported in the literature. Second I describe the method of the study, and afterwards the results, where the freshmen's answers to the diagnostic test are discussed.

THEORETICAL FRAMEWORK

Matz (1980), Linz (1990) and Malle (1993) suggest that the transition from the arithmetical context to the algebraic context is not easy for students, and that difficulties may be due to interpretations in the new context. Liedenberg et al (1998) stated that in arithmetic it is often possible to avoid conventions related to the algebraic structure, while in algebraic expression or equations conventions of hierarchy of operations cannot be evaded. This means that many errors may be due to a deficient interpretation of the order of operations in algebraic expressions and equations.

Malle (1993) studied how students recognize the term structure, without transformation, of the algebraic expression. He asked students to circle sub-terms in an algebraic expression. For example a common error could be, when the expression:

ٛگتٛگ / ڗٛ٠گتٛگ, was is circled as follows:  (3)

The form in which the student circle sub-terms shows that the student has an deficient structure sense, and will probably simplify the expression x^2+y^2 in the numerator and denominator. This is a very common error, which might have other different explanations.

Hoch and Dreyfus (2006, pp.306) distinguish between manipulation skills and structure sense, and present the following definitions:

A student is said to display manipulation skills (MS) if s/he can:
Solve an equation or factor an expression when given explicit instructions.
Substitute correctly in a given formula.

A student is said to display structure sense (SS) for high school algebra if s/he can:
Recognize a familiar structure in its simplest form. (SS1)
Deal with a compound term as a single entity and through an appropriate substitution; recognize a familiar structure in a more complex form:
o where the compound term contains a product or power but no sum.(SS2a)
o where the compound term contains a sum and possibly also a product or power. (SS2b)
Choose appropriate manipulations to make best use of a structure:
o where the structure is in its simplest form. (SS3)
o where the compound term contains a product or power but no sum. (SS3a)
o where the compound term contains a sum and possibly also a product or power.

Analyzing this definition, it is clear that the substitution principle is important. For example; a student is asked to factor the following algebraic expression:

9yٻ 6y 1 (2)

Would s/he be able to factor the expression, if s/he could not identify $9y^2$ as $(3y)^2$? (in terms of Hoch and Dreyfus the student does not recognize a compound term as a single entity; SS2a)

In another factorization like:

$$x^{2} \quad 1 \quad 3x \; x^{2} \quad 1 \qquad (3)$$

If students cannot factorize this expression, it is probably a structure sense problem, SS2b, because they are not able to substitute x^2+1 as a single entity [i.e., $(x^2+1)(1+3x)$].

Having structure sense implies to “see” a “surface” structure and a “hidden” structure (Liedenberg, 1998). Recognizing “hidden” structure is the ability to create an equivalent structure in a simplified form. For example;

$x \quad y \quad w \quad v$; have a surface structure of 4 numbers, and the hidden structure is the sum of two numbers; a + b; where a $\quad x \quad y$, and b $\quad w \quad v$.

Errors might underlie a great variety of reasons, like conjoin errors, errors in the interpretation of the equal sign, view of operation signs, misconceptions with the distribution law, among others (Nolte, 1991; Malle, 1993; Tietze, 2000 and Eccius, 2008). In this study, errors are analyzed in terms of structure sense; the reader might find other interpretations of student’s errors.

METHOD

A 42 items test was administrated to 270 freshmen (Universidad Panamericana, campus Guadalajara, in Business School, July 2010) to investigate their difficulties and errors in understanding basic arithmetic operations, simplifying and transforming algebraic expressions, and solving linear and quadratic equations (Appendix 1). The freshmen come from a great variety of high schools in Guadalajara and of other cities north-west and south-west from Guadalajara. The test was not a multiple-choice test; therefore inference can be made by the written procedures of students, who were asked to show all their work.

The first part of the study is an overall counting of correct answers in the test. The study presents a graph of students’ scores. This quantification of the percentage of correct answers would be a measure, in low percentage items, of the magnitude of the algebraic deficiencies of freshmen, and justifies the need of attending these difficulties.

The important part of this study lies in the analysis of students’ answers. The written procedures of freshmen’s productions give an insight in their structure sense, their conceptions, and misconceptions. Which procedures are related to a deficient structure sense? For the purpose of the paper, procedures with errors that might be related to a lack of structures sense were picked out and reported (some student’s productions were scanned and presented in this paper). For some items, the most common errors were discussed in terms of structure sense. Some freshmen’s conceptions of structure sense are typical or systematic, others are very unique. Not all problems with structure sense can be discussed in this paper.

In accordance to Malle (1993), 25 students were asked to circle sub-terms (Appendix II). Circling terms and sub-terms show very clear student’s structure sense (scanned circle processes were presented). Later, to realize a deeper analysis of students’ conceptions of term structure, 5 students were asked to think loudly. These interviews helped to understand students’ structure sense.

Finally some errors in financial mathematics courses were analyzed in terms of lack in structure sense.

RESULTS

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Part 2: Structure s calculations

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Circling Process

<table>
<tr><td>Interpretation: The circling process in the numerator, shows, that the student understands, that s/he must first multiply w and z, but afterwards s/he interprets a bracket for the expression ύ · ἀ ό; therefore s/he subtracts in the numerator also u. This error in consequence produces the error in the test.</td></tr>
</table>

<table>
<tr><td>Circling process</td></tr>
<tr><td>Circling procedure denotes a problem of structure sense. S/he does not know what to perform first, w is circled twice, in the same level. When students have problems like this, they probably will have errors in manipulating algebraic expressions, because their structure sense is very weak.</td></tr>
</table>

<table>
<tr><td>Circling process</td><td>Probable error in:
M C 1 $\frac{i}{12}$ ٹپ</td></tr>
<tr><td colspan="2">The form in which the student circled the algebraic expression gives an insight in her/his ideas of structure sense. Probably s/he will perform first the power of 3, and then the sum. This might have the consequence of powering i/12 first, and then add 1.</td></tr>
</table>

<table>
<tr><td>Circling process</td><td>Probable difficulties with:
M C 1 $\frac{i}{12}$ ٹپ</td></tr>
<tr><td colspan="2">The student's circle process illustrate a multiplication of 5, before powering the expression (2x+3). The structure sense shown above is related to an order of operation deficiency. Student might multiply C by (1 + i/12), before powering.</td></tr>
</table>

<table>
<tr><td>Algebraic error probably related to structure sense</td><td>Probable difficulties with this equation in finance:
ὠ $\frac{ὠ}{1,03}$ 2500</td></tr>
</table>

Often students do not recognize the structure of this equation. The number 3 does not divide the whole left side of the equation. Being aware of the structure of the equation will help students to manipulate equations of this type.

<table>
<tr><td>Error in the test

Related structure sense:
Not visualization of (x^2+ 1) as an entity,</td><td>Difficulties in other courses in business' schools.
The deduction of compound interest would be of some difficulty for the student, due to the problem to "see" (1+r) as an entity.</td></tr>
<tr><td colspan="2">Hoch and Dreyfus (2006), related this problem, to an incapacity of the student to recognize (x^2+ 1) as an entity. Students will not have the ability to factor the shown expression. As a consequence the deduction of the formula for compound interest will be difficult too.</td></tr>
</table>

<table>
<tr><td>Error in the test
Students were asked to factor the expression:</td><td>Structure sense problem
Even if there is no mathematical error, the expression was not factored. What is for the student a factored expression? Does s/he know the structure of a factored expression?</td></tr>
<tr><td colspan="2">Student was asked if she knows what is meant by a factored expression. She said, she factored "y", because it is a common factor in two of the summands. She does not have a clear idea of a factored expression as the multiplication of factors.</td></tr>
</table>

<table>
<tr><td>Circling processes.</td><td>Error in the test</td></tr>
<tr><td colspan="2">Both circling processes manifest students' first idea of the expression as a fraction. The first process may be the structure sense of the student with the error in the test shown. The second circling process has a double interpretation of structure sense. Fraction is circled, but also the $(x-5)^{m+2}$. This student is not aware of what the structure of this expression is for her/him, which ends probably with a confusion of what to do, when s/he wants to simplify the expression.</td></tr>
</table>

Circling processes	Error in the test

Error in the test have the same sturcture sense problem as the first circling process. Student simplifies a root, with the power of only one summand. In the second circling process, is not clear if the student identifies x^2+3 as an entity.	

Error in the test	Structure sense
	Student's answer to, why he circled as an entity, was: "you have the same in the numerator as in the denominator, so you can cancel them. This structure sense is the same used in the first cancelling process. In the second image of student's work, the student identifies a different structure as it is. S/he does not "see" a plus sign, and afterwards s/he "forgets" the denominator. The last scanned answer registered a different structure. This structure sense lack is related to order of operations, s/he first adds (y+2) and (y+2) and then multiply the result with "y".
In all cases students have a lack in structure sense, therefore they are not able to simplify correctly.	

CONCLUSIONS

The vast majority of freshmen at Business Schools have a low performance in mathematic skills. Their achievement in algebra is hindered by a lack of structure sense.

As a consequence, in general, students cannot succeed in other courses, like finance, business, economics and operations, since they are very concerned dealing with mathematics. Teachers sometimes have to explain mathematics in other subjects.

The lack of structure sense of some freshmen illustrate why they often make errors in simplification and manipulation of algebraic expressions, or in solving equations.

This report makes evident that there must be a systematic effort to identify and quantify student's abilities. Diagnostic assessments will provide teachers and professors information to identify the sources of students' low performance, such as a lack in structure sense, misconceptions and misapplications. It was found that many students have a lack in structure sense and that many errors are related to these deficiencies in recognizing the form of algebraic expressions.

Universities often cannot involve students directly in calculus courses; they have to implement remedial courses in algebra to improve student's mathematic skills. Sometimes beside the remedial courses students need more personal advising and additional hours of practice. Will courses help, if teachers and professors do not know students' problems with structure sense?

Universities have to work closer with high schools and make more emphasis that freshmen have a low performance in mathematics when they enter College. They should transmit these results to high school directors and teachers, emphasizing the importance of detecting misconceptions in structure sense as they appear in school mathematic lessons. If some problems in manipulation of algebraic expressions might be related to the lack of structure sense, teachers should help students to develop structure sense. For example, there are some procedures referred by Malle (1993), to develop the structure sense of students.

REFERENCES

Eccius C., (2008). Mathematikdidaktische Fehleranalysen zur Schulalgebra; Schülerwissen und Lehrerprofessionswissen, Saarbrücken, Deutschland: VDM Verlag Dr. Müller.

Hoch, M., & Dreyfus, T: (2006). Structure sense versus manipulation skills: an unexpected result. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.),*Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 305-312). Prague, Czech Republic: PME.

Kieran, C: (1979) "Children‟s operational thinking within the context of bracketing and the order of operations," In D. Tall (Ed.), *Proceedings of the third international conference for the psychology of Mathematics Education* (pp.128-135). Coventry, UK: Warwick University, Mathematics Education Research Centre.

Liebenberg, R: (1997) "The usefulness of an intensive diagnostic test," In P. Kelsall & M. de Villiers (Eds.), *Proceedings of the Third National Congress of the Association for Mathematics Education of South Africa*: *Vol. 2* (pp.72-79). Durban: Natal University.

Linchevski, L & Livneh, D: 1996, "The competition between numbers and structure". In Puig L, and Gutierrez A (Eds.),. *Proceedings of the 20th International Conference of the International Group for the Psychology of Mathematics Education Valencia, Spain*, 3, 257-264

Linchevski, L., & Livneh, D. (1999). "Structure sense: the relationship between algebraic and numerical contexts". *Educational Studies in Mathematics, 40(2),* 173-196.

Lins, R.L.: 1990, "A framework of understanding what algebraic thinking is". In G. Booker, P. Cobb, and T.N. Mendicuti (Eds.), *Proceedings of the Fourteenth International Conference of the International Group for the Psychology of Mathematics Education*, Mexico City, Mexico, 2, 93-101

Malle, G: (1993) Didaktische Problema der elementaren Algebra; Wiesbaden, Deutschland: Vieweg.

Matz, M: 1980, Building a metaphoric theory of mathematical thought. *Journal of Mathematical Behaviour*, 3, 1, 93-166

Nolte, M: (1991)Strukturmomente des Unterrichts und ihre Bedeutung für das Lernen; Bad Salzdetfurth, Deutschland: Franzbecker

Radatz, H: (1979) "Error analysis in mathematics education," *Journal for Research in Mathematics Education, 10*, 163-172.

Tietze, U,P; Klika, M; Wolpers, H. (Hrsg): (2000) Mathematikunterricht in der Sekudarstufe II, Band 1, 2. Auflage, Germany: Vieweg.

Tirosh, D; Even, R; and Robinson, N. (1998) "Simplifying algebraic expressions: teacher awareness and teaching approaches," *Educational Studies in Mathematics, 35*, 51-64.

Warren, E: (2003) "The role of arithmetic structure in the transition from arithmetic to algebra," *Mathematics Education Research Journal, 15*, 122-137.

APPENDIX 1

Administrated Test

UNIVERSIDAD PANAMERICANA
ESCUELA DE CIENCIAS ECONÓMICAS Y EMPRESARIALES
ACADEMIA DE MATEMÁTICAS
UBICACIÓN DE MATEMÁTICAS

A) Calculate:

1. $25-5\cdot 2+4=$
2. $2+3\cdot\sqrt{3}-4=$
3. $9\cdot 3\div 6\cdot 0=$
4. $-8^2=$
5. $\frac{8}{7}\cdot\frac{14}{2}\cdot\frac{9}{4}=$
6. $\frac{21}{45}\div\frac{7}{15}=$
7. $\sqrt{0.16}=$
8. $\sqrt{\frac{1}{9}}=$
9. $\sqrt[3]{-64}=$
10. $\sqrt{4^2+3^2}=$

B) Simplify the expressions:

11. $10^{-7}\cdot\frac{10^4}{10^{-2}}=$
12. $b^{3x+1}b^{1-3x}=$
13. $\frac{(x-5)^{m+2}}{(x-5)^m}=$
14. $\frac{x^{\frac{3}{2}}}{x^{\frac{2}{5}}}=$
15. $(-3x^{-4})^2=$
16. $\sqrt[3]{\sqrt[6]{y^{36}}}=$
17. $\frac{\sqrt[3]{z^4}}{\sqrt[3]{z}}=$
18. $\sqrt{x^2+y^2}=$

C) Perform and simplify the following algebraic operations:

19. $\frac{25x^{-1}y^4z^{-2}}{-5x^2y^{-1}z^{-4}}=$
20. $14x^2-6xy+8xy-3y^2=$
21. $3\cdot(x-2)+4\cdot(3-x)-(x-5)=$
22. $(z^2-5z+4)\cdot(z-1)=$
23. $\sqrt{x^2+y^2}=$

D) Factorize the algebraic expressions:

24. $z^2-16z+64=$
25. $9y^2+6y+1=$
26. $x^2+1+3x(x^2+1)=$
27. $x^2-12x+35=$
28. $y^3-9y=$

E) Simplify the algebraic fractions:

29. $\frac{(x-5)(x+3)}{x+3}=$
30. $\frac{y\cdot(y+2)+(y+2)}{y+2}=$

31. $\dfrac{(z+4)\cdot(z-1)+2}{z+4} =$

32. $\dfrac{3}{a+b} - \dfrac{2}{a-b} =$

F) Solve the equations:

33. $x\cdot(3-x)-4\cdot(1+x)+x\cdot(x-3)=0$

34. $6-\dfrac{2x}{3}=9$

35. $\dfrac{x+3}{x-4}=0$

G) Solve the quadratic equations:

a) Perform with a factorization:

36. $x^2-4x+3=0$

37. $2x^2+2x=0$

b) Perform using the quadratic formula:

38. $2x^2-x-15=0$

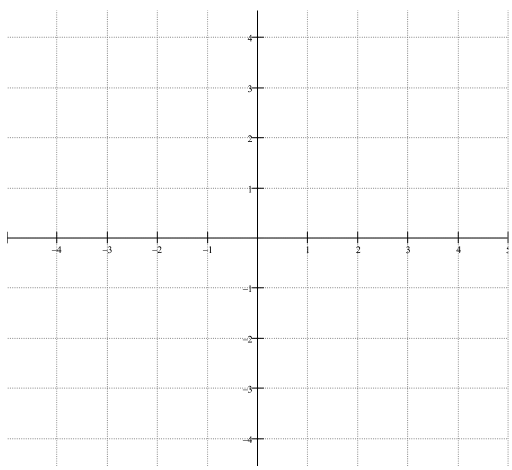
39. $z^2-9=0$

H) Calculate the intersection of the two lines:

40. x – y = 2
y – 2x = 1

I) Graph the function:

41. $y=\dfrac{1}{2}x+1$

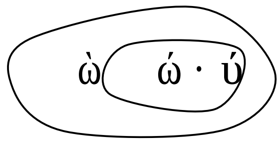


J) An article is labeled $232 including value added taxes. If the value added tax is 16%, what is the price of the article before taxes?

APPENDIX 2

Circle sub-terms and terms in the following algebraic expressions.

On the board the next example was shown, to illustrate students what they have to do:



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