

SAS and the Efficient Portfolio

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ABSTRACT

Given a specified return or a series of returns, this study provides a SAS program to calculate weights for at least a fifty-stock portfolio. Existing SAS portfolios in the public domain provide examples for only two or three stock portfolios. Also, we apply risk aversion measures to maximize investor utility. Users concur that this program is easier to use than the Excel programs for the efficient portfolio.

INTRODUCTION

This paper evolves from one of the authors' need for a SAS¹ program to allocate a portfolio of 50 stocks or greater for an investments class. Given no SAS program in the public domain for a fifty-stock portfolio, we provide senior level and graduate investments and portfolio management students with a hands-on exercise. The Markowitz portfolio is a cornerstone of portfolio management and diversification, but is mathematically complex. One tenet of Markowitz is that portfolios must be diversified. Statman (1987) shows portfolios of 30 or 40 stocks reduce diversifiable risk. However, many institutional equity portfolios have at least 100 stocks. Existing computer programs in the public domain are limited in their usefulness. This paper extends the basic portfolio analysis from the common three stock portfolio examples found in most texts to a more diversified and realistic 50 stock portfolio. Excel spreadsheets such as Stephens (1998) provide efficient stock portfolios, but can invert only a 50 by 50 matrix. Also, Excel spreadsheets are limited to 256 columns of data. In addition, Bodie, Kane, and Marcus (2009, p. 232) report that due to a quirk in Excel software and a peculiarity of the Solver routine, the covariance matrix must

be corrected for bias -- multiply the values times $n/(n-1)$, where n is the number of returns. SAS does not require such adjustments.

With SAS for Windows, advanced finance students can perform matrix calculations in Proc IML (the SAS procedure for interactive matrix language). Many SAS and other programs provide an efficient frontier, but the applications are limited to two or three stock portfolios (See Michaud, 1994). This paper provides a SAS model to capture portfolio weights for fifty stocks or more given a specified return.

The remainder of this paper is organized as follows: first we develop a SAS program without short sale restrictions (the classic Markowitz case) compare results with an Excel version (Stephens, 1998), to include examples of output for a 60 by 10 return matrix. Next, a SAS program for a 60 by 50 return matrix provides portfolio weights. Also, we apply risk aversion measures to maximize investor utility. The resulting programs provide advanced finance students with realistic and statistically viable applications.

REVIEW OF RELATED LITERATURE

Prior to using the SAS program one should review Markowitz (1952), the seminal paper in portfolio theory. Also, investment texts such as Bodie, Kane and Marcus (2009) and Haugen (2001) review the Markowitz model. The Markowitz portfolio objective is to minimize the portfolio variance as shown by:

$$\sigma^2 = \sum_{i=1}^n \sum_{j=1}^n v_i w_j \text{cov}_{ij} \quad (1)$$

$$\text{subject to } \sum_{i=1}^n w_i = 1.$$

Also, the portfolio return is found by:

$$\mu = \sum_{i=1}^n w_i E(r_i) \quad (2)$$

where w_i is the proportion of the portfolio invested in each asset, $E(r_i)$ is the expected return for each asset, and μ is the desired weighted portfolio return.

To effectively use and understand SAS, Excel or other portfolio application students must understand matrix calculations. Waner (2000) briefly reviews matrix calculations. Griffiths, Hill, and Judge (1993) provide matrix calculations in more detail. In matrix notation, equation (1) is:

$$\text{Portfolio variance} = W'V W \text{ subject to } RW = M, \quad (3)$$

where W is an $n \times 1$ vector of portfolio weights, W' is the transpose of W , V is an $n \times n$ variance covariance matrix, R is a $2 \times n$ return matrix, M is a 2×1 matrix that defines the required return.

$$R = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ r_1 & r_2 & \dots & r_n \end{bmatrix} \quad (4)$$

$$M = \begin{bmatrix} 1 \\ \mu \end{bmatrix} \quad (5)$$

The H matrix which results from the Lagrangian multiplier, fixes the mean and variance points on the efficient frontier for a set of n assets (Stephens, 1998).

$$H = RV^{-1}R' \quad (6)$$

R is the $2 \times n$ matrix of column average returns. Matrices cannot be divided, but must be inverted. V^{-1} is the inverse of the variance covariance matrix (V). R' is the transpose of R .

The portfolio weight matrix (W) is an $n \times 1$ column matrix resulting from:

$$W = V^{-1}R'H^{-1}M \quad (7)$$

The resulting weights sum to 1. Since short selling is allowed company weights can be negative.

SAS PROGRAM – UNRESTRICTED TEN STOCK PORTFOLIO

Instead of typing the data directly into the program, it is more efficient to read the return data (infile) from an external file such as an Excel spreadsheet that is converted to a space delimited, formatted text file. The infile allows repeated use of the program by changing only the data file. For a ten stock portfolio the program is:

```
Data Returns;  
infile "e:\RM.prn";  
input x1 x2 x3 x4 x5 x6 x7 x8 x9 x10
```

To calculate in matrix language one must use the SAS procedure for interactive matrix language (proc IML). Once Proc IML is initiated, data must be read into the return matrix (RMAT). This is preferable to typing the 60x10 matrix into the program via the “cards” function.

```
Proc IML;  
use returns;  
read all var {x1 x2 x3 x4 x5 x6 x7 x8 x9 x10} into RMAT;
```

In keeping with standard practice for new computer programs, students should print all input and output for the first iteration. This ensures that data is input correctly and output is calculated correctly.

```
Print RMAT;
```

Next, users should define the number of rows (n) and the number of columns (k). The use of f=1 and g =2 precludes SAS errors of having characters and numbers in the same matrix.

```
f = 1;  
g = 2;  
n = 60;  
k = 10;  
  
/* number of rows*/  
/* number of columns*/
```

The variance/covariance matrix requires a few steps. *RAVG* is the calculated column average returns. The covariance is $COV = (x - \bar{x})(y - \bar{y}) / (n - 1)$. *RMX* is the $(x - \bar{x})$ matrix and *RMX'* is the $(y - \bar{y})$ matrix. The several components of the *RMX* when cross-multiplied and divided by $n - 1$ provide the variance/covariance matrix (*V*). *VINV* is the inverse of the *V* matrix.

```

RAVG = j(f,k,0);
RAVG = RMAT[:,j];      /* to calculate column average returns, a 1 x k matrix*/
Print RAVG;
xbar = REPEAT(RAVG,n,f);      /* to create n x k matrix*/
Print xbar;
RMX = j(n,k,0);
RMX = RMAT-xbar;
Print RMX;
V = j(k,k,0);
V=RMX`*RMX/(n-1);      /* to calculate covariance matrix*/
Print V;
VINV =j(k,k,0);
VINV = INV(V);      /* to take inverse of V*/
Print VINV;

```

The *M* matrix defines the required return. Here we require a 0.5% weekly return.

```

M = {1,0.005};      /* to define M and required return (must change)*/
Print M;

```

The *H* matrix fixes the required return on the efficient frontier. Users need to convert the *RAVG* matrix to a 2 by *k* matrix by vertically combining (concatenating) *DIG*, a 1 x *k* row matrix, with *RAVG*, a 1 x *k* matrix, to obtain *R*, 2 x *k* matrix. (This places *DIG* on top of *RAVG*.)

```

DIG = j(1,k,1);      /* to define 1 X k row matrix of 1s*/
R = j(g,k,0);
R =DIG//RAVG;      /* to concatenate (combine) DIG and RAVG to a 2 x k matrix*/
Print R;
H =j(2,2,0);
H = R*inv(V)*R`;
Print H;

```

The portfolio weight (W) calculation uses elements defined previously, the inverses of V ($VINV$) and H ($HINV$), the transpose of R , and the M matrix.

```
HINV = j(2,2,0);
HINV = INV(H); /* to take inverse of H*/
PRINT HINV;
W = j(1,k,0);
W = VINV*R`*HINV*M; /* to calculate portfolio weights*/
Print W;
```

Once the portfolio weight is obtained, one calculates the portfolio variance by multiplying the transpose of W by V and W .

```
PVAR = j(1,1,0);
PVAR = W`*V*W; /* to calculate portfolio variance given a return and weights*/
Print PVAR;
quit;
```

SAS output for the ten-stock portfolio is shown in Exhibit 1. Portfolio weights range from -0.461429 to 0.8619385. Based on the derived weights, the weekly portfolio variance (PVAR) is 0.000458.

(Insert Exhibit 1 about here)

Exhibit 2 shows the Excel output using the Stephens (1998) procedure. Once the variance/covariance matrix is adjusted by multiplying the values times $n/(n-1)$ -- where n is the number of companies -- the results are identical with the SAS output.

(Insert Exhibit 2 about here)

SAS PROGRAM – UNRESTRICTED FIFTY STOCK PORTFOLIO

For a fifty stock portfolio use the above program, but make the changes in bold type. Some users have trouble in the conversion process from Excel spreadsheet to SAS data. To avoid this problem we split the data into two input files of 25 stocks each.

```

Proc IML;
use returns;
Data Returns;
infile "e:\ RM1.prn " /* to infile the first return matrix*/
input x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18 x19 x20
x21 x22 x23 x24 x25 ;
infile "e:\ RM2.prn " /* to infile the second return matrix*/
input x26 x27 x28 x29 x30 x31 x32 x33 x34 x35 x36 x37 x38 x39 x40 x41 x42 x43
x44 x45 x46 x47 x48 x49 x50;
read all var {x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18
x19 x20 x21 x22 x23 x24 x25 } into RMAT1 ;
Print RMAT1;
read all var {x26 x27 x28 x29 x30 x31 x32 x33 x34 x35 x36 x37 x38 x39 x40 x41
x42 x43 x44 x45 x46 x47 x48 x49 x50};
Print RMAT2;
f = 1;
g = 2;
n = 60; /* number of rows*/
k = 50; /* number of columns*/

```

In this case RMAT must be initialized since it has not been defined. Also, we must combine the two input matrices.

```

RMAT = j(n,k,0);
RMAT = RMAT1 || RMAT2;

```

The remainder of the program is the same as for the ten stock portfolio:

```

RAVG = j(f,k,0);
RAVG = RMAT[:,,]; /* to calculate column average returns, a 1 x k matrix*/
Print RAVG;
xbar = REPEAT(RAVG,n,f); /* to create n x k matrix*/
Print xbar;
RMX = j(n,k,0);
RMX = RMAT-xbar;
Print RMX;
V = j(k,k,0);
V=RMX`*RMX/(n-1); /* to calculate covariance matrix*/
Print V;
VINV =j(k,k,0);
VINV = INV(V);
Print VINV;
M = {1,.005};
Print M;
DIG = j(1,k,1);

```

```

R = j(g,k,0);
R =DIG//RAVG;
Print R;
H =j(2,2,0);
H = R*inv (V)*R`;
Print H;
HINV = j(2,2,0);
HINV = INV(H);
PRINT HINV;
W = j(1,k,0);
W = VINV*R`*HINV*M;
Print W;
PVAR = j(1,1,0);
PVAR = W`*V*W;
Print PVAR;
quit;

```

SAS on line sources help with other troubleshooting.

Exhibit 3 provides SAS output weights for the fifty-stock portfolio. Portfolio weights range from -0.18855 to 0.5140987 and cause a weekly portfolio variance of 0.0000147 which equates to an annual variance of 0.08%. Thus, with a lower variance the 50-stock portfolio shows the benefit of increased diversification.

(Insert Exhibit 3 about here)

SAS PROGRAM –FIFTY STOCK PORTFOLIO WITH UTILITY MAXIMIZED

In this section we expand the SAS program to trace the efficient frontier based on portfolio weights for a range of returns. For each set of portfolio returns we apply the CFA risk aversion measure to maximize investor utility. The equation from Bodie, Kane , and Marcus (2009) is :

$$U = E(R) - \frac{1}{2} A \sigma^2 \quad (8)$$

where U is investor utility, $E(R)$ is the expected portfolio return, A is the risk aversion factor (from 1 lowest to 5 highest), and σ^2 is the portfolio variance. The resulting SAS program follows below.

Modifications to the program in the previous section are shown in bold.

Data Returns;

```

infile "g:\IRM.prn";                                /* to infile the return matrix*/
input x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18 x19 x20 x21 x22 x23
x24 x25 ;
infile "g:\2RM.prn";
input x26 x27 x28 x29 x30 x31 x32 x33 x34 x35 x36 x37 x38 x39 x40 x41 x42 x43 x44 x45
x46 x47 x48 x49 x50;
Proc IML;
use returns;
read all var {x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 x11 x12 x13 x14 x15 x16 x17 x18 x19 x20 x21
x22 x23 x24 x25 } into RMAT1;                        /* to read variables into RMAT1*/
Print RMAT1;
read all var {x26 x27 x28 x29 x30 x31 x32 x33 x34 x35 x36 x37 x38 x39 x40 x41 x42 x43
x44 x45 x46 x47 x48 x49 x50} into RMAT2;
Print RMAT2;
f = 1;
g = 2;
n = 60;                                              /* number of rows*/
k = 50;                                              /* number of columns*/
A = 3/52;
RMAT = j(n,k,0);
RMAT = RMAT1 || RMAT2;
RAVG = j(f,k,0);
RAVG=RMAT[:,,];                                /* to calculate column average returns, a 1 x k matrix*/
Print RAVG;
xbar = REPEAT(RAVG,n,f);                          /* to create an n x k matrix*/
Print xbar;
RMX = j(n,k,0);
RMX = RMAT-xbar;                                /* to calculate RMX*/
Print RMX;
V = j(k,k,0);
V=RMX`*RMX/(n-1);                                /* to calculate covariance matrix*/
Print V;
VINV =j(k,k,0);
VINV = INV(V);                                    /* to take inverse of V*/
Print VINV;
P = 0.0206;
M={1,.0206};                                    /* to define M and required return (must change)*/

```

```

U =0;
Print M;
DIG = j(1,k,1);
R = j(g,k,0);
R =DIG//RAVG;      /* to concatenate (combine) DIG and RAVG to a 2 x k matrix*/
Print R;
H =j(2,2,0);
H = R*inv(V)*R` ;      /* to calculate H*/
Print H;
HINV = j(2,2,0);
HINV = INV(H);      /* to take inverse of H*/
PRINT HINV;
Do until (P<=0);
W = j(1,k,0);
W = VINV*R`*HINV*M;      /* to calculate portfolio weights*/
Print W;
PVAR = j(1,1,0);
PVAR = W`*V*W;      /* to calculate portfolio variance given a return and weights*/
Print PVAR;
U =j(1,1,0);
U = P -0.5*A*PVAR;
PRINT U;
P = P-.0025;
M = M - {0, .0025};
end;
quit;

```

Exhibit 4 shows a graphical representation of an efficient frontier with weekly portfolio returns ranging from 2.06% to zero, utility scores, and portfolio variance for our sample's efficient frontier.

(Insert Exhibit 4 about here)

CONCLUSION

Given exact results for Excel and SAS programs, the SAS program demonstrates its accuracy. In addition, the program allows for portfolio expansion to fifty stocks or more. Users find this SAS program easier to use and takes less time than the Excel spreadsheet. Also, there is less chance for human error in the manipulation of spreadsheet data, especially for the

variance/covariance matrix. The SAS Proc IML guide and Michaud (1994) are useful references. Also, there are several online sources for SAS programs such as *SAS Online Samples*.

Although, SAS programs are efficient, users must be aware of a few potential problems. With any computer program, users must be careful that the input data are correct. One common error is the duplication of rows or columns in the input matrix. This makes the matrix not square and the matrix cannot be inverted.

As shown by the output for ten stock portfolios for SAS (Exhibit 1) and Excel (Exhibit 2), the SAS program provides the same results as the Stephens (1998) process. Thus, with SAS one can obtain weights for any size portfolio. This should enhance student learning and application for realistic portfolios (50 stocks or greater) that maximize diversification. In future research we will add a short sale restriction and position limits.

ENDNOTES

¹ SAS is a registered trademark of the SAS Institute in Cary, NC.

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Exhibit 1: SAS Program Ten-Stock Portfolio Output

Below is the output for the ten stock portfolio, given a 0.5% weekly return. V represents the Variance-covariance matrix. VINV is the inverse of V. W represents portfolio weights. PVAR is the portfolio variance given the weights shown.

V10x10 =

0.0009	0.0002	0.0003	0.00002	0.0004	0.0000	0.0003	0.0004	0.0001	0.0003
0.0002	0.0014	0.0000	0.0000	-0.0000	-0.0000	0.0001	-0.0000	0.0002	0.0001
0.0003	0.0003	0.0000	0.0000	0.0002	0.0000	0.0002	0.0001	0.0001	0.0001
0.0000	0.0000	0.0000	0.0004	0.0001	-0.0001	0.0000	0.0000	0.0001	0.0001
0.0004	0.0004	-0.0000	0.0001	0.0007	0.0001	0.0004	0.0005	0.0002	0.0003
0.0000	0.0000	-0.0000	-0.0000	0.0001	0.0002	0.0001	0.0002	0.0001	0.0001
0.0003	0.0003	0.0001	0.00004	0.0004	0.0001	0.0006	0.0003	0.0002	0.0001
0.0004	0.0004	-0.0000	0.00002	0.0005	0.0002	0.0003	0.0009	0.0001	0.0002
0.0001	0.0002	0.0001	0.00007	0.0002	0.0001	0.0002	0.0001	0.0004	0.0001
0.0003	0.0006	0.0001	0.00007	0.0003	0.0001	0.0001	0.0002	0.0001	0.0003

VINV 10x10 =

2700.7100	-334.8800	-1474.1500	377.6400	-39.4600	1161.9000	-367.3700	-804.1000	182.0100	-1622.8700
-334.8800	857.4300	285.5400	-97.3000	240.3200	110.4600	-124.5800	151.7000	-558.9800	8.6600
-1474.1500	285.5400	4598.9100	103.7400	-1005.2500	-180.9300	-172.6600	738.5800	-829.2900	343.7300
377.6400	-97.3000	103.7400	2661.8500	-957.7400	643.8000	226.9400	184.6500	-412.6300	-406.2000
-39.4600	240.3200	-1005.2500	-957.7400	3690.8500	-466.4700	-1226.1000	-821.6300	164.6200	-1176.2000
1161.9000	110.4600	-180.9300	643.8000	-466.4700	6072.3200	-541.4300	-1220.6700	-654.5700	-875.0900
-367.3700	-124.5800	-172.6600	226.9400	-1226.1000	-541.4300	2948.1400	-214.9400	-877.7900	534.1100
-804.1000	151.7000	738.5800	184.6500	-821.6300	-1220.6700	-214.9400	2193.2700	-227.0200	15.5900
182.0100	-558.9800	-829.2900	-412.6300	164.6200	-654.5700	-877.7900	-227.0200	3914.9500	-368.3300
-1622.8700	8.6600	343.7300	-406.2000	-1176.2000	-875.0900	534.1100	15.5900	-368.3300	5028.7400

Exhibit 2: Excel Markowitz Portfolio Results based on Stephens (1998)

Rmat is the weekly return matrix. Vmat is the adjusted variance/covariance matrix – terms multiplied by $n/(n-1)$, where n is the number of companies. Hmat is the H matrix that fixes mean and variance points on the efficient frontier. Mmat fixes the return at 0.0005. Weights are security weights.

	1	1	1	1	1	1	1	1	1	1
Rmat	-0.0027	0.0008	-0.0020	-0.0011	-0.0005	-0.0002	0.0046	0.0014	0.0009	-0.0013
Covariance Matrix Unadjusted	0.0009	0.0002	0.0003	0.0000	0.0004	0.0001	0.0004	0.0005	0.0002	0.0004
	0.0002	0.0014	0.0000	0.0001	-0.0000	-0.0000	0.0001	-0.0000	0.0002	0.0001
	0.0003	0.0000	0.0004	0.0000	0.0003	0.0000	0.0002	0.0001	0.0001	0.0001
	0.0000	0.0001	0.0000	0.0004	0.0002	-0.0000	0.0000	0.0000	0.0001	0.0001
	0.0004	-0.0000	0.0003	0.0002	0.0007	0.0002	0.0004	0.0005	0.0002	0.0003
	0.0001	-0.0000	0.0000	-0.0000	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001
	0.0004	0.0001	0.0002	0.0000	0.0004	0.0002	0.0007	0.0004	0.0002	0.0002
	0.0005	-0.0000	0.0001	0.0000	0.0005	0.0002	0.0004	0.0009	0.0002	0.0003
	0.0002	0.0002	0.0001	0.0001	0.0002	0.0001	0.0002	0.0002	0.0004	0.0001
	0.0004	0.0001	0.0001	0.0001	0.0003	0.0001	0.0002	0.0003	0.0001	0.0004
Vmat	0.0010	0.0002	0.0003	0.0000	0.0004	0.0001	0.0004	0.0005	0.0002	0.0004
	0.0002	0.0014	0.0000	0.0001	-0.0000	-0.0000	0.0001	-0.0000	0.0002	0.0001
	0.0003	0.0000	0.0004	0.0000	0.0003	0.0000	0.0002	0.0001	0.0001	0.0001
	0.0000	0.0001	0.0000	0.0005	0.0002	-0.0000	0.0000	0.0000	0.0001	0.0001
	0.0004	-0.0000	0.0003	0.0002	0.0008	0.0002	0.0004	0.0005	0.0002	0.0003
	0.0001	-0.0000	0.0000	-0.0000	0.0002	0.0003	0.0002	0.0002	0.0001	0.0001
	0.0004	0.0001	0.0002	0.0000	0.0004	0.0002	0.0007	0.0004	0.0003	0.0002
	0.0005	-0.0000	0.0001	0.0000	0.0005	0.0002	0.0004	0.0010	0.0002	0.0003
	0.0002	0.0002	0.0001	0.0001	0.0002	0.0001	0.0003	0.0002	0.0004	0.0001
	0.0004	0.0001	0.0001	0.0001	0.0003	0.0001	0.0002	0.0003	0.0001	0.0004
Hmat	9497.88	-6.94718								
	-6.95	0.09821								
Mmat	1									
	0.005									
Weights										
	-0.383528									
	0.089007									
	0.025129									
	0.205076									
	-0.461410									
	0.130404									
	0.861939									
	0.184226									
	0.089733									
	0.259424									

Exhibit 3 Fifty-Stock Portfolio Weights based on the SAS Program

W represents portfolio weights for the fifty stock portfolio, given a 0.5% weekly return. PVAR is the portfolio variance given the weights shown.

W	PVAR
0.0820	0.000458
-0.1886	
-0.0732	
-0.0480	
-0.0276	
-0.1153	
-0.0652	
-0.1740	
-0.0458	
0.0111	
0.1188	
-0.1155	
-0.0803	
0.5141	
0.0502	
-0.1387	
-0.1368	
-0.0188	
0.0734	
-0.0553	
-0.0624	
0.1201	
0.1607	
-0.1144	
-0.0025	
0.0812	
0.1292	
0.2294	
0.0888	
0.1098	
0.0730	
-0.1640	
-0.0029	
0.0787	
-0.0705	
0.1375	
-0.0810	
0.1531	
0.0567	
0.0387	
0.0448	
0.2465	
0.1764	
-0.0717	
0.1821	
-0.1069	
-0.0745	
0.0772	
-0.0137	
0.0141	

Exhibit 4: Security Weights and Utility Scores for an Efficient Frontier

