Does the Internal Rate of Return Calculation Require a Reinvestment Rate Assumption?—There Is Still No Consensus

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ABSTRACT

In financial management and related courses, professors teach various capital budgeting methods, including payback period, net present value (NPV) and internal rate of return (IRR). Many textbooks state that the IRR calculation implicitly assumes interim cash flows are reinvested at the IRR. Thus, as an example, if a project has a 25 percent IRR, the calculation implicitly assumes interim cash flows are reinvested at 25 percent, until the end of the project. In contrast, many textbooks state that the NPV method assumes interim cash flows are reinvested at the firm's risk-adjusted weighted average cost of capital (WACC). Many argue that the reinvestment assumption implicit to the NPV calculation is more realistic than the assumption used for the IRR calculation, because the WACC is less than the IRR for a favorable project. However, a thorough review of the textbooks used by professors teaching capital budgeting techniques finds that some textbooks state there is no reinvestment rate assumption made in IRR calculations while many others are silent about the assumption. In addition, there are scholarly articles that argue that the IRR calculation does not require a reinvestment assumption. For example, Keef and Roush (2001) term it the "fallacious reinvestment assumption." The lack of consistency among the textbooks and scholarly literature on such a basic finance concept that is taught to so many students is troubling because it produces confusion. This paper summarizes our findings from current textbooks used in a range of disciplines, revealing the stark inconsistency in the use of this assumption for IRR calculations.

INTRODUCTION

Suppose there were two first grade classes in an elementary school with one teacher instructing the students that 2+2=4, while across the hall the second teacher was instructing the students that 2+2=5. It wouldn't be long before the incompetence of the second teacher was discovered by parents and administrators. When it comes to the presentation of basic material in the classroom, we expect our teachers and their supporting pedagogy to be correct and void of ambiguity.

In the field of financial management, there are few things more fundamental and ubiquitous than the net present value (NPV) and internal rate of return (IRR) calculations. Both are germane to capital budgeting practices used by today's corporate managers. These concepts are taught to thousands and thousands of students in finance, accounting, and economics courses, and not just to business students but also to engineering students. While the cash flows and discount factors required for these calculations involve case-by-case judgment calls, the intrinsic calculations themselves *should not be arbitrary*. We should expect that professors and textbooks are uniform in the underlying steps and the need for assumptions in the calculation. This does not imply that the pedagogical materials must be identical or that professors need to use the same methods to present the concepts; instead, we argue that the underlying steps and the need for assumptions should be consistent. *Our research finds that this is not the case*. Based on the inconsistencies we've uncovered in current, well-known textbooks, the following fictitious vignette illustrates what could be happening on campuses across the country:

At a certain prestigious university Professor Black is teaching his investments class. He is instructing his students how to calculate the yield to maturity of a bond. As he keys the required inputs into his calculator, he carefully explains to his students that the calculation *assumes that the interest payments the bond makes during its lifetime will be reinvested at the yield to maturity*.

Across the hall, Professor White is teaching her corporate finance class. She is instructing her students how to calculate the internal rate of return of a capital project. As she keys the required inputs into her calculator, she carefully explains to her students that the calculation *assumes that the cash flows from the project during its lifetime will be reinvested at the internal rate of return*.

Across the campus in the engineering college, Professor Gray is teaching his engineering economics class. He is instructing his students how to calculate the internal rate of return of a capital project. As he keys the required inputs into his calculator, he carefully explains to his students that the calculation *makes no assumption whatsoever about reinvestment of the cash flows from the project during its lifetime*.

Can they all be right? If not, how many of them are wrong?

The inconsistency we have uncovered occurs with regards to IRR in capital budgeting. So why are we comparing two professors teaching IRR in finance and engineering courses to an investments professor who is teaching yield to maturity (YTM)? Although IRR is applied to capital budgeting and YTM is applied to investment analysis, the IRR and YTM calculations are mathematically identical calculations (see Webster, 2003). Thus, the IRR and the YTM are opposite sides of the same coin. The main difference is that YTM is a measure of return to the providers of debt financing outside the firm, while IRR is a measure of a project's total return inside the firm, which is paid out to the firm's providers of capital, including shareholders, or reinvested in the firm. A corporation's cash flows derive from the projects it invests in and are the source of return for its providers of capital. Webster (2003) uses the YTM concept as a way to introduce the IRR concept. He remarks (p. 520) that "the methodology for determining the yield to maturity on a bond is the same as that used for calculating the internal rate of return."

Our paper is arranged in the following order. In the next section, we discuss key pieces of literature that reveal the inconsistency in the usage of the reinvestment assumption and show what several scholars have said about this topic. In the third section, we present our research methodology. In the fourth section, we discuss our research findings and present some analysis

of the data. In the fifth section, we include a discussion, and then we end with our conclusions and recommendations for future research.

INCONSISTENCY IN THE LITERATURE

Our research follows nearly ten years after the publication of "Discounted Cash Flow Methods and the Fallacious Reinvestment Assumption: A Review of Recent Texts," by Keef and Roush (2001). Their research examined the reinvestment assumption used by contemporary textbooks to explain the conflict between NPV and IRR calculations. The research methodology they used ten years ago is similar to that used in our paper. The primary goal of this research is to see how the textbooks compare today versus ten years ago, to extend the investigation to additional disciplines, and to answer the question: Is the use of the reinvestment assumption as conflicted today as it was ten years ago? In the methodology section, we discuss similarities between our approach and that of Keef and Roush. Regardless of the theoretical stance someone takes on reinvestment, the fact remains: there continues to be an inconsistency in the textbooks, reference books and literature concerning the reinvestment assumption.

One of the earliest papers to clearly reject the reinvestment rate assumption is titled "The Internal Rate of Return and the Reinvestment Fallacy" by Keane (1979). He states "…it is desirable to clarify…issues which are constant sources of confusion," one being that "…neither the IRR nor the NPV method contains any implicit assumptions about the reinvestment rates available for the intermediate cash flows" (p. 49). The debate regarding the validity of the assumption revolves around the conflict that can emerge between the NPV and IRR methods. When looking at the NPV profiles of two projects, the Fisher rate (see Dudley, 1972) is the discount rate where the profiles intersect and NPVs are equal. At discount rates below the Fisher rate, one project's NPV will be higher, while the other project's IRR will be higher. This "conflict" leaves it up to the analyst to decide whether to use the NPV or IRR method to decide the ranking between the two projects. Some argue that the different reinvestment assumptions for NPV versus IRR create this conflict. Yet, Keane's position is that there are other factors to explain this, and he is dismissive of the reinvestment assumption argument. There are others who long ago have shown that there is no implicit reinvestment assumption when calculating IRR (see Alchian, 1955; Beidleman, 1984; Doenges, 1972; Dorfman, 1981; and Dudley, 1972).

The teaching of the reinvestment assumption is not new. As far back as *Corporation Finance*, Schwartz (1964, p. 200) states, "the implicit assumption of the internal rate-of-return formulation is that the fund throw-off over the life of the project can be reinvested at the same rate as that earned by the initial project." Trusting that professors taught what was in their books, this indicates that Dr. Schwartz was teaching students at Lehigh University about the reinvestment assumption nearly half a century ago. Ironically, another Lehigh professor, Carl R. Beidleman, wrote in an article some 20 years later (1984, p. 128) that "the mere suggestion that the models attributed *any* return to the cash flows after release has led to errors in texts and in teaching DCF capital budgeting since its inception." Yet, not only do textbooks today continue to use the reinvestment assumption, but they are not presenting, or even alluding to, the debate that can be found in scholarly articles. For example, in their most recent update to their popular *Fundamentals of Financial Management* textbook, Brigham and Houston (2009, p. 346) state "The NPV calculation is based on the assumption that cash inflows can be reinvested at the project's risk-adjusted WACC, whereas the IRR calculation is based on the assumption that cash flows can be reinvested at the IRR."

The apparent inconsistency, if not confusion, about the importance of making a reinvestment assumption is summarized succinctly by Bierman and Smidt (1993). They acknowledge that "it is frequently claimed that the IRR method assumes reinvestment at the IRR" (p. 92). They believe this claim is "inexact." They argue that the IRR calculation can be computed without making any assumption about reinvestment…and that is correct. When finance professors teach IRR to students, they don't typically consider how the cash flows will be utilized. They could be used to pay dividends; they could be used to invest in new projects; they could be used to purchase financial assets. Yet, Bierman and Smidt go on to say that the cost of the cash used for a project and the value of the cash produced "are necessary to decide if an investment is acceptable." Thus, they make the distinction between the mathematical calculation of IRR and the subsequent use of IRR to make a decision about whether to accept or reject a project. They fall into a different "camp" than others we have read because they are clearly not making a reinvestment assumption that the values of cash produced equal the IRR. Instead, they recognize that some sort of reinvestment assumption would have to be made in some instances in order to value the cash produced and to make a decision about a project. It is not surprising to us to find a more thorough and nuanced discussion of the reinvestment assumption in a capital budgeting textbook than what is typically found in more general finance textbooks that devote less space to any one topic.

A student learning IRR for the first time could be totally confused if they were reading *Engineering Economy* by Sullivan, Wicks and Koelling (2009). The authors assume the reinvestment assumption for NPV calculations, but say that the IRR method "is not limited by this common assumption." They even go on to footnote the Bierman and Smidt (1993) reference that we cited earlier in our paper, which makes it clear that there is no need for a reinvestment assumption when calculating IRR. After making it clear to the reader (likely an unsuspecting engineering student who is seeing material on capital budgeting for the first and only time) that IRR doesn't require a reinvestment assumption, in a later section of the book the authors state "the reinvestment assumption of the IRR method may not be valid in an engineering economy study" (p. 217). They then give an example of a project with an IRR of 42.4 percent and a cost of capital of 20 percent, and observe that "it may not be possible for the firm to reinvest net cash proceeds from the project at much more than 20 percent." In order to solve this problem (and the multiple IRR problem), the authors introduce the external rate of return (ERR) method, which is identical to the MIRR method taught in many finance books. Thus, within the same chapter of this textbook, an engineering student is taught that the IRR calculation doesn't require a reinvestment assumption, but then is taught the ERR method as a remedy to this unrealistic (reinvestment) assumption. How are students supposed to understand capital budgeting with that sort of exposé? The nuance that a student is left to grasp is that IRR does not require any explicit reinvestment assumption in order to calculate IRR. However, as observed by Bierman and Smidt (p. 60), "one might need to know the reinvestment rates to compare alternatives."

While some scholars in academia might believe that the debate over the use of the reinvestment rate assumption for IRR calculations is long over, a quick check of the practitioner literature shows that some in industry are still grappling with the issue. For example, Kelleher and MacCormack (2004), consultants with McKinsey & Company, encourage managers to "avoid using IRR entirely or at least make adjustments for the measure's most dangerous assumption: that interim cash flows will be reinvested at the same high rates of return" (p. 16). They say that this flawed reinvestment assumption "can lead to major capital budgeting distortions" (p. 17). The simple way for practitioners to avoid the problems with IRR is to

discontinue its use. However, as long as those in industry perpetuate its use and business professors continue to teach it, it's highly unlikely that financial managers will abandon its use and solely rely on the NPV methodology. In fact, Graham and Harvey (1999) surveyed Fortune 500 companies and found that the IRR method is the leading evaluation technique among 12 alternatives.

RESEARCH METHODOLOGY

The methodology we used was influenced by the approach taken by Keef and Roush (2001) ten years ago. They examined textbooks in only two areas: management accounting and finance and they applied several constraints to their search: (1) "they were clearly undergraduate texts"; (2) "the texts contained a copyright date of 1995 or later"; (3) "they were available in a New Zealand library." They also rejected books if they did not "explicitly discuss the NPV versus IRR conflict." In our research, we were just as interested to see if MBA textbooks were using the reinvestment assumption as undergraduate texts, so we did not limit ourselves to undergraduate books. Below we discuss the copyright date criterion we used and, of course, we did not limit our search to a New Zealand library because our schools are in the United States.

As for the NPV versus IRR conflict, we did not limit our search in that regard either. Of course, capital budgeting concepts are taught in other courses and not just business courses. For instance, we wonder how many finance professors are aware of the extensive teaching and writing from the engineers on capital budgeting. One unique contribution of our research is that we looked at more disciplines than Keef and Roush in order to obtain a broader sense about the continued use of the reinvestment assumption. McDaniel, McCarty, and Jessell (1988) noted that "many texts in financial management, real estate, engineering economy, etc., state that DCF methods have implicit reinvestment assumptions." The areas we researched were: (1) financial management/managerial finance, (2) corporate finance, (3) capital budgeting, (4) financial and/or managerial accounting, (5) engineering economics, (6) managerial economics, and (7) real estate finance. Our goal was to find at least 15 textbooks in each discipline to raise the confidence level of the statistical testing discussed later in the paper. We decided to look at the financial management and the corporate finance textbooks as two distinct categories, because corporate finance books often contain more advanced material. It is more likely that an MBA corporate finance textbook, or even an undergraduate-level corporate finance text, would discuss the reinvestment assumption than an introductory financial management text. You would expect that authors of books with more basic treatment of finance principles would be more inclined to sidestep the reinvestment rate assumption topic to avoid confusing students.

Textbooks in each of the disciplines were located at the libraries or ordered through the Interlibrary Loan systems at Kutztown University of Pennsylvania and The Pennsylvania State University. Also, there were other ways to obtain access to the books; for example, we teach finance courses at our respective schools and have various textbooks sent to us by publishers for our review. Keef and Roush (2001) uncovered a dramatic inconsistency among finance and managerial finance textbooks, so we were interested to see if the textbooks today are any more uniform on the reinvestment assumption. Their sample included 48 books with a copyright date of 1999 or earlier, 11 books with a copyright of 2000, and just one book with a copyright date of 2001. Their paper was submitted in May 2000 for peer review, thus explaining why the majority of books had earlier copyrights. Evidently, as the paper went through the review and editing phase prior to publication, this gave the authors time to update their list of books with more

current textbooks. Because we wanted to see what the textbooks since the Keef and Roush paper were using as assumptions, we limited our research to books with copyright dates after 2000.

We reviewed each textbook to see first if it presented the NPV and IRR concepts; if so, we then checked for an explicit mention of the reinvestment rate assumption, as it pertains to these calculations. We had two research assistants helping us with this project.[1] In order to obtain consistent information on each textbook, we used a common checklist for screening the books, shown in Appendix 1. We categorized the findings into (1) "yes," the text discusses and uses the reinvestment assumption for the IRR calculation, (2) "no," the text discusses the assumption, but does not use it, and (3) the text is "silent" about the reinvestment rate assumption. If a book was silent about the assumption, we made no assumptions as to why. An author could leave out the assumption because he does not believe it is necessary or a decision has been made to exclude it from the book for brevity reasons. Here we deviate from the approach taken by Keef and Roush (p. 111) as they "assume that silence on the reinvestment assumption is an act of commission rather than an act of omission." They make the argument that "it is not necessary to deny a false proposition."[2] It is our belief that an author could agree with the assumption, but decide to exclude it from his book and leave it for discussion in more advanced textbooks. Thus, we did not make an absolute assumption that silence was an act of commission or omission.

RESEARCH FINDINGS AND ANALYSIS

We examined a total of 64 textbooks across seven disciplines, with the breakdown shown in Table 1. Most of the books (22) are finance books, with roughly half of them covering financial management and the other half covering corporate finance. The complete list of books by discipline is provided in Appendix 2. Capital budgeting topics are most closely associated with finance courses, but they are by no means exclusive to business students. For instance, many engineering students are taught these concepts in their engineering curriculum. Typically, these courses are taught by engineering faculty using textbooks written by engineering professors. For example, one of the well-known engineering economics professors is Chan S. Park of Auburn University. He is the author of *three* textbooks in the area of engineering economics. Consistent with having engineering faculty teach and write textbooks on capital budgeting techniques, the engineering disciplines have a journal titled *The Engineering Economist* as an outlet for scholarly research related to capital budgeting and other finance-related topics.

Table 1: Does the textbook use the reinvestment assumption?

Discipline	"Yes"	"No"	"Silent"	Total
Financial Management & Managerial Finance	8	0	2	10
Corporate Finance	6	3	3	12
Capital Budgeting	3	1	2	6
Financial and/or Managerial Accounting	5	0	4	9
Engineering Economics	3	5	7	15
Managerial Economics	4	0	4	8
Real Estate	1	0	3	4
Totals	30	9	25	64

Each book in Table 1 presents the IRR methodology and was categorized as "yes" it uses the reinvestment assumption, "no" it does not use it, or "silent" if it says nothing about the assumption. For example, we obtained 12 corporate finance textbooks with copyright dates after 2000. Of those books, 6 state that the IRR calculation utilizes the reinvestment rate assumption; 3 state they do not use it; and 3 are silent regarding the assumption. Earlier, we quoted from Brigham and Houston (2009) to provide an example of what is said in support of using the assumption. To provide an example of an opposing view, below we quote from Ross, Westerfield and Jordan (2008), authors who do not invoke the reinvestment assumption:

We *will* take a stand on one issue that frequently comes up in this context. The value of a project does not depend on what the firm does with the cash flows generated by that project. A firm might use a project's cash flows to fund other projects, to pay dividends, or to buy an executive jet. It doesn't matter: How the cash flows are spent in the future does not affect their value today. As a result there is generally no need to consider reinvestment of interim cash flows.

Table 2: Frequency of reinvestment assumption found by Keef and Roush and in Our Research

	Keef and Roush Results		
Discipline	**"Yes"**	**"No"**	**Total**
Finance	24	11	35
Managerial Accounting	19	6	25
Totals	43	17	60
	Our Results		
Discipline	**"Yes"**	**"No"**	**Total**
Finance	14	8	22
Managerial Accounting	5	4	9
Totals	19	12	31

Source: Keef and Roush (Table 3, p. 111, 2001)

In Table 2, we report the findings from Keef and Roush's (2001) paper. They found (p. 111) that 43 out of 60 textbooks, or 72 percent of the books, used the reinvestment assumption. Their research categorized books differently than ours in that they used "yes" when the "text clearly implied that the reinvestment assumption is the cause of the NPV versus IRR conflict." We simply looked to see if the textbook used the assumption. Typically the authors would state the assumption without providing the motivation for it. Nevertheless, in order to do a more apples-to-apples comparison to the data collected by Keef and Roush, we combined our finance books into one category. Likewise, we changed the heading from "Financial and/or Managerial Accounting" to "Managerial Accounting," as none of the textbooks we surveyed was strictly a financial accounting textbook. The final change we had to make was to shift the "silent" books into the "no" column, in essence applying the same assumption as Keef and Roush regarding silence. These changes in our data enabled us to put our data into Table 2 using the identical configuration used by Keef and Roush and to ask the question: Are more or less authors utilizing the reinvestment rate assumption now as compared to that found ten years ago? Simple

inspection of the data indicates that fewer are using the assumption today, but can this be verified statistically?

CHI-SQUARE TESTS OF INDEPENDENCE

Based on the data that we collected for this study, one question we had was the following: Is the decision to use the reinvestment assumption dependent on discipline? For example, are the finance authors utilizing the assumption more so than the engineering economists? Simple inspection of the data in Table 1 indicates that the answer is "yes," the portion of books using the assumption is dependent on discipline, but can we verify that statistically?

Chi-square test of independence – Walker-Check-Randall data from Table 1

H_0: discipline and reinvestment assumption are independent
H_1: discipline and reinvestment assumption are not independent
$\chi^2_{calculated} = 22.07$, $\chi^2_{critical} = 21.03$, i.e., significant at $\alpha = 0.05$

We reject the null hypothesis and conclude that there is a statistically significant dependency between academic discipline and treatment of the reinvestment assumption in the data from which Table 1 is a sample.

We also conducted a chi-square test of independence on the Keef-Roush data. We found there to be no dependency on discipline. Thus, this might explain why the earlier authors restricted their analysis to these two disciplines.

Chi-square test of independence - Keef and Roush data from Table 2

H_0: discipline and reinvestment assumption are independent
H_1: discipline and reinvestment assumption are not independent
$\chi^2_{calculated} = 0.40$, $\chi^2_{critical} = 2.71$, i.e., *not* significant at $\alpha = 0.10$

We fail to reject the null hypothesis and conclude that there is no statistically significant dependency between academic discipline and treatment of the reinvestment assumption in the data from which Table 2 is a sample.

DISCUSSION

The impetus behind this area of research is the feedback one of the coauthors of this study received on a research paper submitted to *The Engineering Economist* in 2007. In the manuscript, the reinvestment assumption was used in the analysis. A reviewer stated that this was "theoretically incorrect" and felt strongly enough about this to recommend the paper be rejected for publication. When we did our literature search, we found that the engineers have already grappled with the validity of the reinvestment assumption and seem to have resolved this issue years ago. In the engineers' world, this issue is an old one that apparently no longer needs discussion. In a paper published in *The Engineering Economist*, Lohmann (1988) claims that he

has illustrated "the fallacy of the 'reinvestment rate assumption' numerically, mathematically, and intuitively." Not surprisingly, we see that the highest percentage of texts that are either silent or refute the use of the assumption is in the area of engineering economics. In fact, 80 percent of the authors are either silent or say "not needed" to the use of this assumption, while the next closest discipline to take this stance is real estate at 75 percent. Why is there less of a consensus among finance professors? Do the engineers know something we don't know or vice versa?

In this debate about whether the IRR analysis uses or doesn't use an implicit reinvestment assumption, the focus is generally on the inconsistency between the NPV and IRR methodology when selecting between two mutually exclusive projects. At a discount rate below the Fisher rate, the NPV will point to one project as being the best, while IRR will point to the other. Prior scholars observe that this inconsistency is a result of two different reinvestment assumptions—NPV assumes reinvestment at the cost of capital, while IRR assumes reinvestment at the IRR. It was not this debate about the proper reinvestment rate assumption that piqued our interest in whether or not the IRR calculation requires the reinvestment rate assumption. Instead, we approach this issue by drawing a parallel to fixed-income investing and the analytics used in that area.

Earlier, in the introduction of our paper, we told a vignette of a professor teaching YTM to his investments class. The YTM calculation on a bond takes into account coupon interest and any capital gain (or loss). Fabozzi (2006) observes that the YTM also "considers the interest-on-interest component, although implicit in this computation is an assumption that the coupon payments can be reinvested at the computed yield to maturity." Fabozzi observes (p. 140) that if the coupon interest payments are not reinvested at the YTM, "the actual yield realized by an investor will be greater than or less than the yield to maturity." Note that it is possible to reinvest coupons at a higher or lower yield than the YTM, so the realized yield could actually be higher than the "promised yield" (another term for YTM). We have been unable to uncover any source that refutes the need to make a reinvestment assumption when calculating YTM.

If an investor on the outside of the firm is exposed to a realized bond yield that is potentially higher or lower than the promised yield because of uncertain reinvestment rates on coupon payments, it follows (doesn't it?) that a corporate manager could be exposed to a realized return on a project that is potentially higher or lower than the IRR because of uncertain reinvestment rates on the project's cash flow. In Table 3, we show three projects with identical IRRs. If management's investment criterion is based on IRR, which project is "best" for the corporation? All three projects have identical investments of $5 million, equal lives, but different cash flows. Project 1 has a constant cash flow; project 2 has cash flows that grow at 20 percent per year; and project 3 has a lump sum at the end of the project's life. Based on IRR, the firm should be indifferent between these three projects. It is conceivable that a firm is indifferent to the cash flow differences. Yet, our analysis finds that project 1 has the greatest reinvestment risk. When we used a 12 percent reinvestment assumption, project 1's return drops the most, to 16.26 percent. If the IRR calculation does not require an implicit reinvestment assumption, which some scholars argue, then it seems that project analysis might require another return metric to fully assess a project—one that *does* require a reinvestment rate assumption. In the world of fixed-income investing, the YTM calculation conveniently serves both purposes. Why that isn't the case with IRR calculations (in the minds of some authors) is a debate for future research.

Table 3: Cash flow comparisons of three projects

Cash Flows	Project 1	Project 2	Project 3
CF_0	(5,000,000)	(5,000,000)	(5,000,000)
CF_1	1,671,619	1,416,267	0
CF_2	1,671,619	1,557,893	0
CF_3	1,671,619	1,713,683	0
CF_4	1,671,619	1,885,051	0
CF_5	1,671,619	1,073,556	12,439,015
IRR	20.0%	20.0%	20.0%
Realized Return	16.3%	16.5%	20.0%

CONCLUSIONS

So where do we stand ten years later? Keef and Roush (p. 110, 2001) stated that "Our assessment of the literature since 1979 is that there has not been a robust denial of the fallacy of the reinvestment assumption." Based on our research, we can make the same assertion ten years later. The use of the reinvestment assumption is found online (see Investopedia.com) and it is found in many of the current textbooks being used by faculty at countless colleges and universities. While there has been debate on the validity of the assumption, we believe that future research should attempt to resolve the dispute once and for all. The assumptions are just as important as the calculations themselves, and academics and practitioners need to come to agreement as to "best practices." Not so long ago, Adler (2006, p. 4) wrote, "The assumptions related to DCF are increasingly becoming so disconnected from business reality that its continued use should come with the following warning, 'This financial management technique is hazardous to your business.'"

Earlier we identified Bierman and Smidt (1993) as making the distinction between the mathematical calculation of IRR and the usage of IRR. They present a convincing argument for employing the reinvestment assumption when deciding whether or not a project is acceptable. The purpose of teaching IRR is not to teach students the IRR calculation per se. Rather, faculty are presenting IRR as another capital budgeting tool to be used by corporate managers. If a professor subscribes to Bierman and Smidt's argument, then he should utilize a textbook that explains the importance of the reinvestment assumption. It follows, then, that textbooks should also discuss the concept of reinvestment risk in the context of capital budgeting and project analysis. Our research found that most finance textbooks *do* discuss reinvestment along with IRR, but that most engineering economics textbooks do not. Perhaps the engineering professors should send their students to the college of business when it comes time to teach their students capital budgeting.

Many different disciplines teach IRR and NPV concepts, including management accounting and managerial economics. They aren't just taught in finance courses. Our research found that there is still significant inconsistency between books when it comes to using the reinvestment rate assumption. Moreover, there is lack of consistency across disciplines. Finance books fall at one end of the continuum with 64 percent using the assumption while the

engineering economics books fall at the other end with just 20 percent using the assumption. Why can't all the disciplines come to an agreement? Elementary school teachers all agree that 2+2=4, so why can't professors agree that IRR does or does not involve the implicit assumption that all interim cash flows can be reinvested at the IRR? We can understand ongoing debate and controversy over more advanced theories and calculations, such as the capital asset pricing model, but IRR calculations are too basic and ubiquitous for this degree of inconsistency. In Appendix 3 we discuss a "capital budgeting paradigm" that provides a starting point for building a consensus.

APPENDIX 1

Textbook Title _____

Textbook Author(s) _____

Textbook Publisher/Location_____

Textbook Edition _____ Copyright Year _____

NPV – pages _____ and attach copies

1. Is the NPV methodology presented?
2. If yes to (1), do they state the reinvestment rate assumption?
3. If yes to (2), what is the specific assumption?
4. Is there any discussion or debate about the validity or need for making a reinvestment rate assumption in order to do the NPV calculations?

IRR – pages _____ and attach copies

5. Is the IRR methodology presented?
6. If yes to (5), do they state the reinvestment rate assumption?
7. If yes to (6), what is the specific assumption?
8. Is there any discussion or debate about the validity or need for making a reinvestment rate assumption in order to do the IRR calculations?

APPENDIX 2

A. Financial Management/Managerial Finance

1. Besley, S., & Brigham, E. F. (2005). *Essentials of managerial finance* (13th ed.). Mason, OH: Thomson Higher Education. (Y)
2. Block, S. B., & Hirt, G. A. (2008). *Analysis for financial management* (9th ed.). New York: McGraw-Hill/Irwin. (Y)
3. Brigham, E. F., & Houston, J. F. (2009). *Fundamentals of financial management: Concise edition* (6th ed.). Mason, OH: South-Western Cengage Learning. (Y)
4. Brigham, E. F., & Houston, J. F. (2004). *Fundamentals of financial management* (10th ed.). Mason, OH: South-Western Cengage Learning. (Y)
5. Brigham, E. F., & Ehrhardt, M. C. (2008). *Financial management: Theory and practice* (12th ed.). Mason, OH: Thomson South-Western. (Y)
6. Brigham, E. F., & Daves, P. R. (2007). *Intermediate financial management* (9th ed.). Mason, OH: Thomson South-Western. (Y)
7. Foerster, S. R. (2003). *Financial management: a primer*. New York: W. W. Norton & Company, Inc. (S)
8. Gitman, L. J. (2003). *Principles of managerial finance brief* (3rd ed.). Boston, MA: Pearson Education, Inc. (Y)
9. Higgins, R. C. (2009). *Analysis for financial management* (9th ed.). New York: McGraw-Hill/Irwin. (S)
10. Lasher, W. R. (2008). *Practical financial management* (5th ed.). Mason, OH: Thomson Higher Education. (Y)

Do the books use the reinvestment assumption? (8 Yes; 0 No; 2 Silent; 10 Total)

B. Corporate Finance

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Do the books use the reinvestment assumption? (6 Yes; 3 No; 3 Silent; 12 Total)

C. Capital Budgeting

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Do the books use the reinvestment assumption? (3 Yes; 1 No; 2 Silent; 6 Total)

D. Financial and/or Managerial Accounting

1. Garrison, R. H., Noreen, E. W., & Brewer, P. C. (2006). *Managerial accounting* (11th ed.). Boston: McGraw-Hill/Irwin. (Y)
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Do the books use the reinvestment assumption? (5 Yes; 0 No; 4 Silent; 9 Total)

E. Engineering Economics

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2. Bowman, M. S. (2003). *Applied Economic Analysis for Technologists, Engineers, and Managers* (2nd ed.). Upper Saddle River, NJ: Pearson Education, Inc. (S)
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Do the books use the reinvestment assumption? (3 Yes; 5 No; 7 Silent; 15 Total)

F. Managerial Economics

1. Adams, J. & Juleff, L. (2003). *Managerial economics for decision making.* New York: Palgrave MacMillan. (S)
2. Baye, M. R. (2009). *Managerial economics and business strategy* (6th ed.). New York: McGraw-Hill/Irwin. (S)

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7. Thomas, C. R., & Maurice, S. C. (2008). *Managerial economics* (9th ed.). New York: McGraw-Hill/Irwin. (Y)
8. Webster, T. J. (2003). *Managerial economics: Theory and practice*. San Diego, CA: Academic Press. (Y)

Do the books use the reinvestment assumption? (4 Yes; 0 No; 4 Silent; 8 Total)

G. Real Estate

1. Brueggeman, W. B., & Fisher, J. D. (2005). *Real estate finance and investments* (12th ed.). New York: McGraw-Hill/Irwin. (S)
2. Ling, D. C., & Archer, W. R. (2008). *Real estate principles: A value approach* (2nd ed.). New York: McGraw-Hill/Irwin. (Y)
3. Floyd, C. F., & Allen, M. T. (2005). *Real estate principles* (8th ed.). Chicago, IL: Dearborn Real Estate Education. (S)
4. Ratcliffe, J., Stubbs, M., & Keeping, M. (2009). *Urban planning and real estate development*. New York: Routledge. (S)

Do the books use the reinvestment assumption? (1 Yes; 0 No; 3 Silent; 4 Total)

Grand Total: Do the books use the reinvestment assumption? (30 Yes; 9 No; 25 Silent; 64 Total)

APPENDIX 3
WALKER-CHECK–RANDALL
CAPITAL BUDGETING PARADIGM

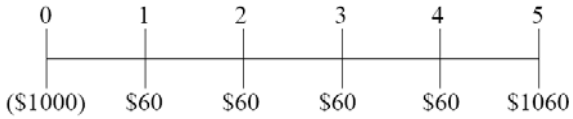
The purpose of this paper is to update the treatment of the reinvestment assumption in finance textbooks, and to extend that work to additional disciplines. However, having read dozens of texts and dozens of papers on the topic, it is difficult to merely report our findings without reporting our conclusions. Dante Alighieri wrote “The hottest places in Hell are reserved for those who in times of great moral crises maintain their neutrality.” Perhaps there is a special place in Heaven reserved for those who observe controversy and provide consensus. Since our conclusions are peripheral to our main purpose they are presented in this Appendix.

The lists of authors on both sides of this issue are long and impressive. We observe that the two sides of the issue, in many ways, are talking past one another, i.e., not arguing consistent circumstances. There are *ex-ante* and *ex-post*, *implicit* and *explicit*, and *internal* and *external* interpretations given to the reinvestment assumption. Even authors who argue vehemently against any reinvestment assumption will discuss how invested funds "grow" at this rate or that.

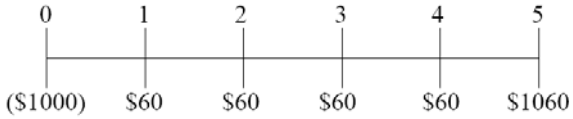
We are greatly swayed by the following example.

Suppose a municipality[3] is planning to finance the construction of a toll bridge by selling a municipal bond. The bridge will require an initial investment of $1000, will produce net cash inflows of $60 per year for five years, and will be sold after five years for $1000. The municipal bond will have a $1000 par value, a five-year maturity, pay 6 percent annual coupons annually, and be sold at par value.

The time line for the municipality to use in evaluating the financial impact of the bridge is:



The time line for an investor considering the purchase of the municipal bond is:



Clearly the internal rate of return (IRR) of the bridge is 6 percent. When the investor calculates *ex-ante* the expected yield to maturity (YTM) on the municipal bond he/she will *implicitly* assume that the intermediate cash flows will be reinvested at the yield to maturity, and the calculated yield to maturity is 6 percent. If the municipality and the investor are using the same calculator, the key strokes used to calculate the IRR and the YTM are identical. If we were to calculate the modified internal rate of return (MIRR) of the project we would *explicitly* use a reinvestment rate of 6 percent and again using similar key strokes on our calculator find that the MIRR = IRR = YTM = 6 percent.

While few would argue against the reinvestment assumption being *implicit* in the YTM calculation and no one could argue that there wasn't an *explicit* reinvestment rate used to calculate the MIRR, many would argue there is no reinvestment assumption in the IRR calculation. Therefore, we suggest the following definition of capital budgeting and a framework for internalizing all intermediate cash flow circumstances.

Capital budgeting is a process which examines the effect of a financial transaction on the transactor's wealth when the transaction's cash flows are sufficiently distributed in time as to be affected by the time value of money. Depending on the method used, the effect on wealth may be expressed as a dollar change, a rate of return on investment, or merely as an increase or decrease in wealth.

Capital budgeting can be applied *ex-ante* to proposed projects as a decision methodology on whether the projects should be undertaken, or *ex-post* to projects already concluded to compare how the assumptions made prior to the project match what actually occurred.

When capital budgeting is applied *ex-ante* it relies on numerous assumptions regarding future events and conditions. For a corporation[4] examining a capital project, some of those assumptions are the:

- amount and timing of the initial investment;
- amounts and timing of any changes in revenue;
- amounts and timing of any changes in explicit costs;
- amounts and timing of any changes depreciation expense;
- applicable tax rate(s);
- lifetime of the project;
- after-tax cash flow from the disposal of assets at the end of the project;
- change in net working capital at the end of the project;
- effect the project has on the riskiness of the firm;
- amount of uncertainty in all of the above; and
- intended disposition of cash inflows during the project.

Regarding the disposition of cash inflows during the project:

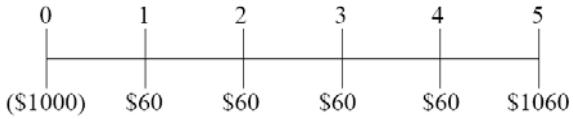
(1) If the corporation knew the intended disposition of cash inflows occurring during the project, then the cash flows resulting from that intended use should be included in the capital budgeting process.

(2) If the corporation knew that cash inflows occurring during the project were to be left idle, then all cash inflows should be assumed to occur at the end of the project when they become available for other use.

(3) If the corporation knew that cash inflows occurring during the project were to be discarded (thrown away) when they occurred, then the analysis should include an equal and concurrent cash outflow and the rate of return on such a project would always be –100%.[5]

(4) If no specific knowledge exists regarding the disposition of cash inflows occurring during the project, then the capital budgeting process must include some assumption, explicit or implicit, regarding that disposition.

For example, assume a corporation is examining a project with cash flows that just happen to be the same as the municipality in our earlier example.



The corporation plans to discard cash flows 1 and 2, allow cash flow 3 to lie idle until the end of the project, and to reinvest cash flow 4 at 20 percent. Then, the future value of the inflows would be:

$$FV_5 = (\$60 - \$60) + (\$60 - \$60) + \$60(1 + .00)^2 + \$60(1 + .20)^1 + \$1060 = \$1192$$

and the project's IRR would be 3.58%.

If instead of planning to discard cash flow 1 the corporation made no explicit assumption about how cash flow 1 would be employed, then the mathematics used to calculate the project's IRR would implicitly assume reinvestment at the IRR. Thus, the future value of the inflows would be:

$$FV_5 = \$60(1+IRR)^4 + (\$60 - \$60) + \$60(1+.00)^2 + \$60(1+.20)^1 + \$1060 = \$1192 + \$60(1+IRR)^4$$

and the project's IRR would be 4.80%.

ENDNOTES

[1] We thank Zach Phipps and Trevor Shelbo for their assistance with this research project.

[2] See Spranca, Minsk, and Baron, 1991, for research that examines the difference in meaning between "commission" and "omission."

[3] We use a municipality in our example to avoid the issue of taxes. Using a tax-paying corporation and a taxable debt instrument complicates but does not invalidate the example.

[4] We return to the more relevant corporate example and assume all cash flows are after-tax and that the tax rate is known with certainty. In practice corporations would need to utilize an expected tax rate.

[5] This curious procedure is required to address criticisms of the reinvestment assumption such as Lohmann, 1988.

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