THE GLOBAL HOMOGENEITY OF INSIDE INFORMATION

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ABSTRACT

This paper finds that claim prices in prediction markets, a new genre of financial markets, follow a Poisson distribution. The significance of this finding is that as soon as a claim in a prediction market is created and thereafter flushes out expert and inside information from around the world regarding that particular claim, claim prices immediately begin forming bell-shaped distributions, implying global agreement regarding the probabilities of claims being realized. This is an interesting finding, implying a surprisingly high degree of global homogeneity of inside information in predictions markets, even though such information is scattered in disconnected and secretive pockets around the world. This finding could also imply that cultural diversities do not significantly affect the interpretation of information in prediction markets.

INTRODUCTION

Prediction markets, a relatively recent innovation, have proven to be remarkably accurate forecasters of future events. Prediction markets – also known as "decision markets" – are online global markets for betting that, according to one comprehensive study by Credit Suisse First Boston, make "uncannily accurate" predictions in finance, economics, business, politics, government, science, sports, natural disasters, culture, and virtually every other realm of human and natural activity. (*Wall Street Journal*, July 30, 2003, p. C1.) In a recent survey of the accuracy of predictions in decision markets, Moyer (2007) found that, "Prediction markets are the best known forecasters of future events, no matter what the event is." (p. 69.) For readers unfamiliar with prediction markets, see Wolfers and Zitzewitz (2004) or Ray (2006) for a history, overview and description of such markets. Readers desiring to see a prediction market in actual operation can insert the phrase "prediction markets" or "decision markets" into an internet search engine such as Google.

Empirically, the predictive accuracy of such markets has been well documented. Berg, Nelson, and Rietz (2008), Cherry (2007), Rosenbloom and Notz (2006), Servant-Schreiber, *et al* (2004), and others have empirically found these financial markets to possess a remarkable ability to accurately forecast future events of all types. Even prestigious scientific journals such as *Science* have examined these markets and found that they possess a "remarkable performance" to make accurate predictions of all types of future events. (*Science,* Feb. 9, 2001; Pencock, *et al*.) The growing body of both empirical and anecdotal evidence attesting to the forecasting ability of prediction markets is quite impressive. Readers can utilize an academic search engine such as ABI/INFORM or Google Scholar to access other studies empirically documenting the remarkable forecasting abilities of this new genre of financial markets.

In practical usage, Microsoft, Hewlett Packard, Intel, Eli Lilly, Google, Corning, Yahoo, and many other major firms use *internal* prediction markets for capital-budgeting purposes in order to predict project revenues, project costs, project completion times, and myriad other capital-budgeting variables. (An internal prediction market is one which is expressly created for a particular firm, and is used only by that firm. If the company doesn't have the in-house expertise to create and operate an internal prediction market, prediction markets such as newsfutures.com will do so for a fee.) In the government sector, NASA uses internal prediction markets to schedule shuttle-launch dates; and, the U.S. Pentagon has created prediction markets to identify and pre-empt acts of terrorism. See Ray (2009, 2008) for a discussion of how firms and government agencies use prediction markets to forecast critical variables for planning purposes. Even the World Economic Forum, held every year in Davos, Switzerland, annually uses these financial markets to assess the probabilities of future global events (terrorism, nuclear war, flu epidemics, financial crises, etc.,) in order to guide its discussions and thereafter formulate its global planning.

One still unresolved question of prediction-market forecasts is, What is the probability distribution of claim prices (which reflect the market's estimated probability of a claim actually coming true) in a prediction market? Over time, as the number of claim prices for a given event grows (as the news changes, and traders react to the news), what type of distribution do these claim prices follow? The first moment of this distribution will determine the mean forecast since the claim was first made. The second moment will determine the variance around this mean (the smaller the variance, the more certain the market is about the forecasting accuracy of the claim price, and vice versa for a larger variance). The third moment, the skewness of the distribution, will determine the non-randomness of the forecasts (distributions skewed to the right have a higher probability of being realized; distributions skewed to the left have a lower probability of being realized). And the fourth moment, the distribution's kurtosis, will determine the highest probability so far assigned to a particular forecast.

The next section of this paper models prediction markets within the context of the Efficient Markets Theory. Thereafter, the price of a one-dollar claim in a prediction market is equated to the probability of that particular claim coming true (a well-known property of prediction markets). Then, the most common types of distributions in the social sciences are examined for their potential ability to adequately model a distribution of claim prices. Finally, the paper deduces that the prices of claims in a prediction market (equivalent to the market's estimated probabilities of the claim coming true) follow a Poisson distribution. Interpretations regarding prediction markets are then derived from the properties of Poisson distributions.

MODELING PREDICTION MARKETS

Prediction markets can be easily modeled within the context of the Efficient Markets Theory:

$E_m\ (\rho_{j,\,t}) = (P_{j,t} \mid \theta_t)$ (1)

where E_m = the market's expectation at time m;

$\rho_{j,\,t}$ = the probability, at time t, of claim j being realized;

$P_{j,,t}$ = the price of claim j at time t; and,

θ_t = the information set utilized by the market at time t to form its expectation.

For recent overviews of the theory of efficient markets, see Malkiel (2003) and Shiller (2003). Because these financial markets operate in cyberspace, they are completely unregulated and easily trade on inside information which, of course, is illegal in regulated markets. Thus, prediction markets are, by definition, among the most efficient markets (and perhaps *the* most efficient market) in the world, being efficient in the weak, semi-strong, and strong forms simultaneously.

The key variable in equation (1) is $P_{j,,t}$ which, as noted, is simultaneously the price of claim j at time t, and also the market's estimated probability, at time t, of claim j being realized. This well-known property of prediction markets can easily be derived by recognizing that a winning claim, which pays one dollar (or some multiple of one dollar) has a probability of 100%, while a losing claim, which pays nothing, has a probability of 0% -- a property commonly referred to as an Arrow Debreu claim. Thus, it follows that the intervening values are the probabilities of the particular claim coming true. (Since most claims trade on a bid/ask basis, the mid-point of the spread is generally used as the probability estimate.) Nobel Economics Laureate William Sharpe (1995) has shown that prices, in conditions such as prediction markets, are, in fact, probabilities.

THEORY vs. EMPIRICISM FOR CLAIMS DISTRIBUTIONS

From the outset, it's important to note that the distributions of claims in a prediction market can only be derived in theory; they cannot be empirically verified. This is so because as a new claim price electronically materializes, it replaces the previous claim price, and prediction markets do not keep data bases of historical claim prices. An updated claim price simply "disappears" electronically and is replaced by the most current electronic claim price. Nevertheless, it would be both interesting and helpful, in understanding prediction markets, to know what type of distribution that claim prices follow. The purpose of this paper is to ascertain that distribution.

SOME LIKELY CANDIDATES FOR FORECAST DISTRIBUTION

A beginning likely candidate for the distribution of claims in a prediction market would be some member of the Family of Stable Distributions (the Cauchy, Levy, and Gaussian distributions), which are the most commonly utilized distributions to explain behavior in the social sciences.

Because the moments of the Cauchy distribution are undefined, the variance of a Cauchy distribution is infinite:

$$\mathrm{E}(X^2) \propto \int_{-\infty}^{\infty} \frac{x^2}{1+x^2}\,dx = \int_{-\infty}^{\infty} dx - \int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \infty - \pi = \infty.$$

(equation 3)

and thus this distribution is inapplicable to a distribution whose second moment is finite, as is the second moment of claim distributions (with boundaries of 0% and 100%).

The Levy distribution's *n*th moment is defined as:

$$m_n \stackrel{\text{def}}{=} \sqrt{\frac{c}{2\pi}} \int_0^\infty \frac{e^{-c/2x}\, x^n}{x^{3/2}}\, dx \tag{4}$$

where $\boldsymbol{c}$ is the scale parameter, and where $\boldsymbol{m_n}$ diverges for all n > 0 so that the moment-generating function of the Levy distribution does not exist. Thus, a distribution with a non-existent second moment is, again, inapplicable to a claim distribution whose second moment is clearly bounded.

With respect to the Gaussian distribution, it's possible that, in theory, a large number of claim prices could approximate a Gaussian distribution. By using the Central Limit Theorem, Samuelson (1972) has shown that, for a distribution with a large number of variables (such as stock prices, which number in the thousands), the lognormals of the subject variables approximate a Gaussian distribution. However, very few claim distributions have such a large number of claims (certainly not, with few exceptions, numbering in the thousands as stock prices do). Most distributions in a prediction market have a relatively smaller number of claims, sometimes as many as just a few for unusual claims, and thus claim prices cannot not follow a Gaussian distribution. Therefore, the entire Family of Stable Distributions can be eliminated as possible candidates of claim-prices distributions. The next section of this paper identifies the parameters needed for a type of distribution to correctly describe a distribution of claim prices.

DISTRIBUTION REQUIREMENTS FOR CLAIM PRICES

The requirements of a probability distribution to correctly describe a distribution of claim prices are as follows:

1. The distribution must consist of discrete positive integers (0, 1, 2, …).
2. The probability values of the distribution must describe the market's estimated probabilities of an event coming true (0%, 1%, 2%, …).
3. The news causing claim prices must occur in a fixed period of time.
4. Each claim must be independent of the occurrence since the last claim.
5. The claims must exist during a fixed period of time.

These requirements are the exact requirements for a Poisson distribution, commonly used to describe random behavior of occurrences of events in fixed time intervals. Requirement #1 above is simply the nature of a Poisson distribution, and simultaneously is the nature of claim prices. Requirement #2 describes the probabilities of random events, such as when people arrive at an elevator (a very common example used to demonstrate the utility of a Poisson distribution). This has an exact analogue in when new claim prices arrive in a prediction market. Requirement #3 describes how the news creates new claim prices during the fixed period of the claim period. Requirement #4 describes prediction-market claims because the news arrives randomly in a fixed time period and is independent of previous news. Requirement #5 is the fixed length of time during which a claim exists.

Because claim prices in a prediction market meet the exact requirements to be a Poisson distribution, it can be logically deduced that claim prices follow a Poisson distribution in a prediction market. Claim distributions can thus be modeled as a Poisson distribution, whose properties are as follows.

Let *k* be a random but *relevant* news announcement (i.e., one creating a new claim price) occurring during the life time of the claim. If the expected number of *k* is λ, then the probability that there are exactly *k* occurrences of λ (*k* being a non-negative integer), and thus k occurrences of claim prices in a given distribution, is equal to:

$$f(k;\lambda) = \frac{\lambda^k e^{-\lambda}}{k!}, \qquad (5)$$

where

- *e* is the base of the natural logarithm (*e* = 2.71828...)
- *k* is the number of relevant news announcements regarding a particular claim, and thus also the number of occurrences of the claim price as effected by *k*
- *k*! is the factorial of *k*
- λ is a positive real number, equal to the average number of relevant news announcements that occur during the given time interval during which a claim exists.

(Note: any probability can be expressed as a non-negative integer by simply multiplying the probability by a base index. For example, a probability of 42% can be expressed as an integer by multiplying it by a base index of 100. (i.e.., 42% X 100 = 42.) As long as all the probabilities of a distribution are multiplied by the same base index, the shape and properties of the distribution do not change. For a more complete discussion of equation (5), see Good (1996).

The parameter λ is not only the mean number of occurrences of *k*, but also its variance:

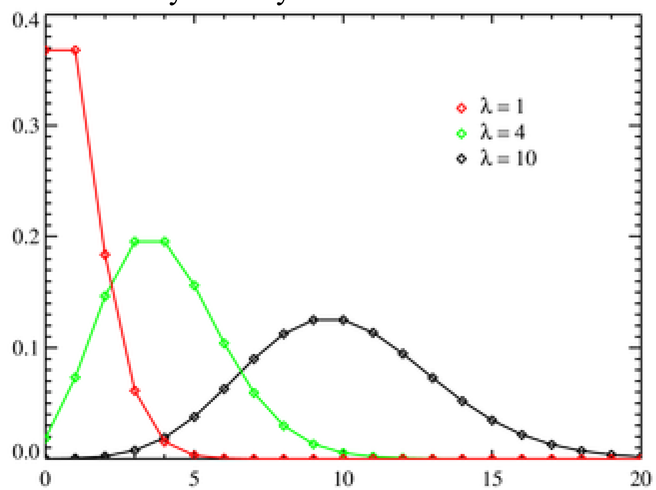
$$\lambda = \sigma_k^2 = \langle k^2 \rangle - \langle k \rangle^2 \qquad (6)$$

Thus, the distribution of claim prices fluctuates about its mean λ with a standard deviation $\sigma_k = \sqrt{\lambda}$. (These fluctuations are commonly referred to as “Poisson noise.”) Furthermore, the skewness of the distribution is $\lambda^{-1/2}$ and its kurtosis is λ^{-1} as derived from the Poisson moment-generating function:

$$\exp(\lambda(e^t - 1)) \qquad (7)$$

As a Poisson distribution, the probability density function of claim prices can be plotted as:

Probability Density Function of Claim Prices



The horizontal axis is *k*, the number of claim prices (which can be extended to any number), while the vertical axis is the probability of *k* (which can be extended to a value of l.0 if market conditions so warrant). (Note: The graph is only defined at integer values of *k* – called "empty lozenges"; the connecting lines are only guides for the eye.) As can be seen, when claim prices follow a Poisson distribution, the distribution becomes bell-shaped after only a few claim prices are made. Virtually all claim prices have at least two or three past claim prices, and so it can be logically deduced that claim prices follow a Poisson distribution. As can also be seen, the Poisson distribution will even accommodate a single claim price (as sometimes occurs for a highly unusual claim).

SUMMARY AND CONCLUSIONS

This paper finds that claim prices in prediction markets follow a Poisson distribution. The significance of this finding is that as soon as a claim in a prediction market is created and thereafter flushes out expert and inside information from around the world regarding that particular claim, claim prices immediately begin forming bell-shaped distributions, implying global agreement regarding the probabilities of claims being realized, with the second moment of the distribution indicating the degree of agreement. This is an interesting finding, implying a surprisingly high degree of homogeneity of inside information in predictions markets, even though such information is scattered in disconnected pockets across the globe. Moreover, this finding could also imply that cultural diversities do not significantly affect the interpretation of information in prediction markets.

REFERENCES

Berg, J., Nelson, F., and Rietz, T. "Prediction Market Accuracy in the Long Run." 2008; International Journal of Forecasting, Spring/Jun 2008, Vol. 24, No. 2, p. 283+.

Cherry, S., "Bet On it!" 2007; IEEE Spectrum, Vol. 44, No. 9, p. 48+.

Good, J., "Some Statistical Applications of Poisson's Work." 1996; Statistical Science. May, 1996. Vol. 1, No. 2, 157-170.

Malkiel, B., "The Efficient Market Hypothesis and Its Critics," 2003; Journal of Economic Perspectives, 17(1), 59-82.

Moyer, M., "How to Predict the Future." 2007; Popular Science, 271(1), 69-71.

Pennock, S., Lawrence, C., Giles, L. and Nielsen, F., "The Real Power of Artificial Markets." 2001; Science, 291(5506), 987-988.

Ray, R. "Planning with Prediction Markets." Strategy. March, 2009. Issue 19, 28-30.

_____. "Decision Markets: A Powerful Capital-Budgeting Tool." July/August, 2008. Corporate Finance Review, 13 (1), 5-9.

______. "Prediction Markets and the Financial Wisdom of Crowds." Journal of Behavioral Finance, 2006; Vol. 7, No., 2-4.

Rosenbloom, E., and Notz, W., "Statistical Tests of Real-Money Versus Play-Money Prediction Markets," 2006;. Electronic Markets, Vol. No, 1, p. 63+.

Samuelson, P., "The Mathematics of Speculative Prices." 1972; Reprinted in SIAM Review, 15, 1-42.

Servan-Schreiber, e., Wolfers, J., Pennock, D., and Galebach, B., "Prediction Markets: Does Money Matter?" 2004; Electronic Markets, Vol. 14, No. 3. p. 243+.

Sharpe, W., "Nuclear Financial Economics," 1995; in Risk Management: Challenges and Solutions by Bever, W., and Parker, G. (McGraw Hill.)

Shiller, R., " From Efficient Markets Theory to Behavioral Finance." 2003; Journal of Economic Perspectives, 17(1), 83-104.

Wolfers, J., and Zitzewitz, E., "Prediction Markets." 2004; Journal of Economic Perspectives, Vol. 18, No. 2 (2), 107-132.