Capital Structure Decision-Making with Growth: An Instructional Class Exercise

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ABSTRACT

This paper offers an instructional class exercise of the capital structure decision-making process. The exercise applies four gain to leverage (G_L) equations including a recent G_L equation that ties together the plowback-payout choice and the debt-equity choice. The latter G_L equation is given by the recent Hull [2010] Capital Structure Model (CSM) with growth. Given estimates for the costs of capital, tax rates and growth rates, this equation can guide managers of growth firms on how to choose an optimal debt level. This paper's exercise demonstrates the interdependency of the plowback-payout and debt-equity decisions when maximizing firm value. By incorporating growth through use of a plowback ratio, this paper extends the non-growth capital structure exercise of Hull [2008, JFEd]. This growth extension has proven to be successful in helping advanced business students understand the impact of the plowback and debt choices on firm value.

INTRODUCTION

Along with capital budgeting, working capital management, and dividend policy, capital structure is one of major topics taught in financial management courses. The capital structure question is: "*How much debt (if any) is needed to optimize firm value?*" If firm value can be negatively affected from the wrong debt choice, it becomes imperative to answer this question. The recent instructional exercise by Hull [2008] attempted to address this question by comparing gain to leverage (G_L) results using three perpetuity G_L equations. These equations come from the G_L models supplied (i) by Modigliani and Miller [1963], referred to as MM, (ii) by Miller [1977], and (iii) by the Capital Structure Model (CSM) of Hull [2007]. The Hull [2008] instructional exercise was able to show students how the incorporation of costs of borrowing through use of the CSM can cause firm value to peak at one debt level choice before falling. The MM and Miller equations are devoid of costs of capital and cannot demonstrate that an interior optimal debt-to-equity ratio exists.

The three G_L equations used in the Hull [2008] exercise were all for a non-growth situation where the firm's operating cash flows are fixed. A recent theoretical extension of the CSM by Hull [2010] has incorporated growth, thus making possible an instructional exercise of capital structure decision-making that concomitantly considers growth as rendered through the plowback-payout choice. In this paper, we use the CSM with growth to extend the non-growth exercise of Hull [2008] by allowing a firm's operating cash flows to grow at a constant rate. In the process, this paper's class exercise ties together the plowback-payout and debt-equity choices illustrating the interdependency of these two choices.

Motivation

This paper is motivated in the same way that prompted the Hull [2008] instructional exercise. The motivation lies in research [Leland, 1998; Graham and Harvey, 2001] that suggests capital structure decision-making cannot properly be taught because it lacks an adequate equation compared to well-accepted equations used to teach capital budgeting and the cost of capital. Simply put, the textbook models for capital budgeting (NPV, IRR, Payback, and PI) and costs of capital (WACC, DVM, and CAPM) are considered more reliable that G_L equations. Without reliable G_L equations, students and practitioners are left to only guess what a firm's optimal debt choice might be. To overcome this "equation" problem, CSM equations were developed by Hull [2005, 2007, 2010] to extend the MM [1963] and Miller [1977] G_L equations by including cost of capital variables capable of capturing more than a tax effect.

Hull [2010] has recently written that the prior perpetuity G_L research has failed to analyze the relation between a firm's plowback-payout choice and its debt-equity choice. To overcome this problem, Hull [2010] broadens the CSM framework by incorporating the plowback-payout choice. In the process, Hull coins and develops a number of new concepts including equilibrating unlevered and levered growth rates. These rates are used by Hull to get growth-adjusted discount rates that are needed to derive his G_L equation with growth.

Growth-adjusted discount rates work in a fashion similar to the Dividend Valuation Model (DVM) with growth where a perpetual cash flow is divided by a discount rate minus a growth rate. However, unlike the DVM where the growth rate is not differentiated for unlevered and levered equity, the CSM with growth can only be derived after growth rates for both unlevered and levered equity are first developed. The theoretical development of the unlevered and levered equity growth rates enable this instructional paper to extend the Hull [2008] pedagogical application so that educators are now offered a method of teaching capital structure decision-making that will be applicable to firms with a plowback ratio greater than zero.

Learning Outcomes

By experiencing the exercise in this paper, upper level business students with a sound background in corporate finance concepts should be able to experience the advanced intricacies of the capital structure decision-making. The following learning outcomes should result. *First*, students should learn how to compute four perpetuity G_L equations and compare these equations based on the different variables that each equation includes in its computation. These variables include tax rates, costs of borrowing, and growth rates. *Second*, for a growth firm, students should learn how the plowback-payout ratio choice affects the optimal debt-to-equity choice including how the combination of great growth and great leverage leads to great risk.

Remainder of Paper

The rest of the paper is organized as follows. The next section reviews capital structure models focusing on the four perpetuity G_L equations used in this paper. The following section contains our instructional exercise of the capital structure decision-making process. The last section provides final remarks including supplementary teaching considerations. The appendices provide solutions for instructors when using this paper's exercise.

GAIN TO LEVERAGE MODELS

Capital structure research is abundant and multifaceted [Modigliani and Miller, 1958; Harris and Raviv, 1991; Myers, 2001; Mahrt-Smith, 2005; Hennessy and Whited, 2005; Strebulaev, 2007; Berk, 2010; Matsa, 2010; Korteweg, 2010]. This paper focuses on one aspect of this research: perpetuity G_L equations originating in the MM [1963] capital structure model.

MM and Miller Equations for G_L

Assuming a non-growth situation, MM assert that the gain to leverage (G_L) is:

$$G_L = T_C D \tag{1}$$

where T_C is the effective corporate tax rate, $D = \frac{I}{r_D}$, I is the perpetual cash flow paid to debt owners, and r_D is the cost of debt. For MM, r_D is the risk-free rate (r_F). Equation (1) disregards personal taxes and leverage-related effects including those associated with financial distress.

Miller [1977] broadens equation (1) by examining the impact of debt from an investor's view after the payment of personal taxes on equity and debt income. The Miller equation is:

$$G_L = [1 - \alpha]D \tag{2}$$

where $\alpha = \frac{(1-T_E)(1-T_C)}{(1-T_D)}$, T_E and T_D are the effective personal tax rates paid, respectively, by equity and debt owners, and now $D = \frac{(1-T_D)I}{r_D}$ with r_D determined endogenously and $r_D > r_F$. At the firm level, G_L is zero for Miller when equation (2) is used. This is because the influence from personal taxes offsets the positive corporate tax shield effect while bankruptcy costs are considered inconsequential.

Empirical research [Warner, 1977; Altman, 1984; Kayhan and Titman, 2007] provides no perfect consensus concerning Miller's claim that leverage-related effects are insubstantial. Some researchers offer specific numbers concerning the positive effect of debt. For example, Graham [2000] estimates that the corporate and personal tax benefits of debt can increase firm value by as little as 4.3% with a mean incremental net benefit of 7.5%, while Korteweg [2010] finds that the net benefit of leverage averages 5.5% of firm value. Hull [2005] finds it to be 5.72% for his real world case study of a gas and electric company. Hull [2010] suggests values ranging from 6.5% to 32.1% depending on the mix of growth and leverage. However, values under 10% are more likely to occur given that firms cannot sustain high growth rates for long periods. Hull suggests there is one caveat with attaining high net benefits from high leverage combined with high growth: these high benefits come with extreme risk.

Theoretically, post-MM researchers favor optimal (or trade-off) capital structure models. Beginning with Baxter [1967], Kraus and Litzenberger [1973], and Jensen and Meckling [1976], earlier optimal theorists argued that G_L is maximized only when a further issuance of debt does not cause the incremental wealth benefits of debt to be greater than its incremental costs. Current theorists [Hennessy and Whited, 2005; Leary and Roberts, 2005; Korteweg, 2010] continue to advance this notion. However, direct and indirect costs from bankruptcy and agency effects (as discussed by optimal advocates) are numerous and arguably impossible to identify and measure with precision for all possible leverage-related effects. The CSM research by Hull [2005, 2007, 2010] attempts to circumvent the problem of trying to measure the numerous direct and indirect agency-bankruptcy effects presented by optimal models. The CSM research does this through equations that require financial managers to estimate tax, borrowing, and (if applicable) growth rates.

CSM Non-Growth and Growth Equations for G_L

Like MM and Miller, the non-growth CSM focuses on an unlevered firm issuing perpetual debt to retire equity. For this non-growth situation, Hull [2007] shows:

$$G_L = \left[1 - \frac{\alpha r_D}{r_L}\right] D - \left[1 - \frac{r_U}{r_L}\right] E_U \tag{3}$$

where

α, r_D, and D are as defined previously when describing equation (2);

r_U is the exogenous cost of unlevered equity with $r_U > r_D$;

r_L is the endogenous cost of levered equity with $r_L > r_U$; and,

E_U (or V_U) is the unlevered equity value for a non-growth firm and equals $\frac{(1-T_E)(1-T_C)C}{r_U}$ with $C = (1-PBR)(CF_{BT})$ where PBR is the before-tax plowback ratio and CF_{BT} is the uncertain perpetual before-tax cash flow generated by operating assets (with PBR = 0 when there is no growth).

The derivation of equation (3) can result in a positive G_L due strictly to designing security types that are collectively more valued by investors as reflected in a lower overall cost of borrowing. This can translate into a higher per share value for equity when $G_L > 0$ with the higher value representable by an additional perpetuity cash flow stream. Hull [2010] labels this additional cash flow stream as "G" and refers to it as enigmatic. He offers a way of computing G arguing that its calculation is important due to its influences on the growth rate of levered equity (g_L). Using g_L as his key concept, Hull [2010] extends (3) so that G_L with growth is:

$$G_L = \left[1 - \frac{\alpha r_D}{r_{Lg}}\right] D - \left[1 - \frac{r_{Ug}}{r_{Lg}}\right] E_U \tag{4}$$

where

α, r_D, and D are as defined previously;

$\mathbf{r_{Ug}}$ is the growth-adjusted discount rate on unlevered equity given as $r_{Ug} = r_U - g_U$ with r_U as the unlevered cost of equity and g_U as the unlevered equity growth rate;

$\mathbf{r_{Lg}}$ is the growth-adjusted discount rate on levered equity given as $r_{Lg} = r_L - g_L$ with r_L as the levered cost of equity and g_L as the levered equity growth rate; and,

E_U (or V_U) is the unlevered equity value for a growth firm and equals $\frac{(1-T_E)(1-T_C)C}{r_{Ug}}$ with C defined earlier when describing equation (3) and now $C < CF_{BT}$ because $PBR > 0$.

Another expression for C is $C = POR(CF_{BT})$ where POR is the payout ratio. PBR and POR are defined on a before-tax basis with $PBR + POR = 1$.

Hull [2005] offers a proof for an equation similar to (4) but, unlike Hull [2010], does not incorporate PBR in his formulation and also does not define g_L in terms of G. In Table 2 of our instructional exercise, formulas for computing g_U and g_L will be given with g_L influenced by G. Finally, as discussed by Hull [2010], equation (4) is the most general equation as it reduces to (3) when growth rates are zero, just as equation (3) reduces to (2) if differences in costs of capital are ignored and to (1) if personal tax rates are also ignored.

INSTRUCTIONAL EXERCISE

Given the above background, we now offer an instructional exercise that extends the Hull [2008] application. The instructional exercise asks students to compute G_L values predicted by MM [1963], Miller [1977], Hull [2007], and Hull [2010]. Through these computations, students can learn how the inclusion of key financial variables influences firm value when managers determine their optimal plowback-payout and debt-equity ratios.

Below, instructors will find six sets of questions for this paper's exercise. At their discretion, instructors may omit questions to condense the exercise or tailor it to fit the degree of difficulty that is desired. Besides the answers provided in Appendices 1−6, Excel spreadsheets with all solutions are available on request. For the convenience of those who have used the prior non-growth pedagogical application of Hull [2008], we use (where applicable) the same values for variables used in this prior exercise. An instructor who has used the non-growth application will observe that Questions 1−2 repeat much of materials in the Hull [2008] exercise while adding a few changes. For brevity's sake, an instructor will also notice that costs of borrowings are now given whereas Hull [2008] had students compute these values using the CAPM. Values for variables in this paper's exercise can be easily modified when using the provided Excel spreadsheets if instructors want to choose values for variables based on their own beliefs. For example, one can modify one cell for PBR or T_C in a spreadsheet and see how a change in this variable's value affects the maximum G_L and the optimal debt choice.

Table 1. MM and Miller Values

[Note. When different, the MM and Miller values are denoted in subscripts.]

$\mathbf{T_{EMM}}$ = personal tax rate on equity income = **0%**	$\mathbf{T_{EMiller}}$ = **5.00%**
$\mathbf{T_{DMM}}$ = personal tax rate on debt income = **0%**	$\mathbf{T_{DMiller}}$ = **15.00%**
$\mathbf{T_C}$ = corporate tax rate = **30.00%**	$\boldsymbol{\alpha}_{\mathbf{Miller}} = \frac{(1-T_{EMiller})(1-T_C)}{(1-T_{DMiller})}$
$\mathbf{V_{UMM}} = \frac{(1-T_{EMM})(1-T_{CMM})C}{r_U}$ where $C = (1-PBR)(CF_{BT})$	$\mathbf{V_{UMiller}} = (1-T_{EMiller})V_{UMM}$
$\mathbf{D_{MM}} = P(V_{UMM})$ where P = proportion of V_{UMM} retired by D_{MM}	$\mathbf{D_{Miller}} = (1-T_{EMiller})D_{MM}$
$\mathbf{G_{LMM}} = T_C(D_{MM})$	$\mathbf{G_{LMiller}} = [1-\alpha_{Miller}]D_{Miller}$
$\mathbf{r_U}$ = cost of capital for unlevered equity = **11.00%**	$\mathbf{r_F}$ = risk-free rate = **5.00%**
I = Interest = $r_D(D)$ where I = 0 for an unlevered firm because D = 0	
$\mathbf{CF_{BT}}$ = perpetual before-tax cash flow generated by operating assets = **$1,654,135,338.34**	
PBR = plowback ratio used on CF_{BT} (PBR = 0 with no growth)	**POR** = payout ratio = 1 − PBR
RE = before-tax retained earnings = PBR(CF_{BT}) with RE = **$0** for no growth when PBR = 0	
C = before-tax cash to equity = (1 − PBR)(CF_{BT}) with C = CF_{BT} for no growth when PBR = 0	

Question 1: Computing MM and Miller Values

Unlevgrowth Inc. (UGI) is an unlevered growth firm. UGI's managers believe it can increase its equity value by retiring a proportion of its outstanding equity through a new debt issue. UGI will treat its new debt as perpetual since it plans to continuously roll it over whenever it reaches maturity. Besides increasing its value through the use of debt, UGI is also considering expansion through its technological innovation. The expansion will involve a new line of marketable products for which future patents will assure constant long-term growth in cash payable to equity owners. Before it considers any valuation impact from a growth decision, UGI wants to consider the impact from issuing debt to retire equity if it maintains its current non-growth situation.

For its first task, UGI's managers want to compute the valuation impact using the G_L equations supplied by MM and Miller. To complete this task, UGI's managers estimate values for variables when using the MM and Miller G_L equations. These values are included in Table 1.

(a) Answer the below questions using the MM viewpoint and assumptions.

(i) From the information in Table 1, what is MM's unlevered equity value (V_{UMM})?

(ii) What is the dollar amount of the MM debt (D_{MM}) that will be issued if UGI retires 0.5 of its unleveraged firm value (e.g., retires 0.5 of V_{UMM})?

(iii) What is the MM gain to leverage (G_{LMM}) from retiring 0.5 of V_{UMM}?

(iv) What is the MM percentage increase in firm value ($\%\Delta V_{MM}$) from retiring 0.5 of V_{UMM} where $\%\Delta V_{MM}$ is G_{LMM} / V_{UMM} in percentage terms?

(v) What is the MM debt-to-firm value ratio (D_{MM} / V_{LMM}) after retiring 0.5 of V_{UMM} (where $V_{LMM} = V_{UMM} + G_{LMM}$.)

(vi) Consider an unlevered firm with no debt. Is the perpetuity cash flow generated from operating cash flows identifiable in terms of an accounting variable? For example, how might CF_{BT} in Table 1 be like earnings before interest and taxes (EBIT)? How might it be like earnings before taxes (EBT) or net income (NI) or dividends paid to equity owners (DIV)? Explain.

(b) Answer the first five questions from (i) through (v) in part (a) except now use the Miller values that consider an investor's viewpoint after they pay personal taxes? In other words, what are $V_{UMiller}$, D_{Miller}, $G_{LMiller}$, $\%\Delta V_{Miller}$, and $D_{Miller} / V_{LMiller}$?

(c) UGI's managers decide it is better to look at other debt choices besides just 0.5. In particular, UGI wants MM and Miller G_L values for nine choices that retire from 0.1 to 0.9 of its unlevered equity with increasing increments of 0.1 as shown in Exhibit 1. Using Excel (or a similar software), repeat parts (a) and (b) for the debt choices given in Exhibit 1 so you can fill in Exhibit 1's blank cells. Is there an optimal leverage ratio? Explain.

Exhibit 1. MM and Miller Values for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P = Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
$D_{MM} = P(V_{UMM})$	1.0526								9.4737
G_{LMM}	0.3158								2.8421
$\%\Delta V_{MM}$	3.00%								27.00%
D_{MM} / V_{LMM}	0.0971								0.7087

$D_{Miller} = P(V_{UMiller})$	1.0000								9.0000
$G_{LMiller}$	0.2176								1.9588
$\%\Delta V_{Miller}$	2.18%								19.59%
$D_{MM} / V_{LMiller}$	0.0979								0.7526

[Note to Exhibit 1. While answers placed in Exhibit 1 can be rounded off, you should not round-off until after you have performed all computations; otherwise, errors can occur. Values for the "0.1" and "0.9" columns have been put in Exhibit 1 to help jump-start the process. Students can copy in the answers from parts (a) and (b) for the "0.5" column.]

Question 2. Computing CSM Values without Growth

UGI is not satisfied with the results from MM and Miller models because it believes its predicted debt choice is unrealistic. Thus, UGI's managers decide to turn to the Capital Structure Model (CSM) without growth. This CSM no-growth equation is: $G_L = \left[1 - \frac{\alpha r_D}{r_L}\right]D - \left[1 - \frac{r_U}{r_L}\right]V_U$. Before using the CSM, UGI estimates the costs of capital (r_D and r_L) for each debt choice. The values for r_D and r_L are given in Exhibit 2. The CSM non-growth value for V_U and D in Exhibit 2 are the same as Miller's V_U and D values because the CSM (like Miller) also considers both personal and corporate taxes. Answer the below questions.

(a) Fill in the blank cells in Exhibit 2. Identify and comment on the debt choice for UGI's maximum G_L and maximum V_L, the largest increase in its firm value (as given by the "%ΔV" row), and the optimal D / V_L.

(b) Explain the significance of the "Incremental ΔG_L" and "Incremental %ΔV" rows and what their first negative values indicate.

Exhibit 2. CSM (without growth) Values for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P = Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
V_U (or E_U)	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
$D = P(V_U)$	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000
r_D	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%	8.18%	5.30%	10.28%
r_L	11.12%	11.36%	11.84%	12.50%	13.28%	14.30%	15.50%	11.36%	18.44%
G_L	0.5361								1.0400
V_L	10.5361								11.0400
E_L	9.5361								2.0400
%ΔV	5.36%								10.40%
Incremental ΔG_L	0.5361								–0.0877
Incremental %ΔV	5.36%								–0.79%
D / V_L	0.0949								0.8152

[Note to Exhibit 2. While answers placed in Exhibit 2 can be rounded off, you should not round-off until after you have performed all computations; otherwise, errors can occur. Values for the "0.1" and "0.9" columns are given to jump-start the computational process. To compute values for any column, begin by computing G_L using the CSM no growth equation. Next, compute V_L given by $V_L = V_U + G_L$. E_L is given by $E_L = V_L - D$. The percentage change in firm value (%ΔV) is G_L as a percentage of V_U. The "Incremental ΔG_L" is the change in G_L from the previous column's G_L value. For the "0.1" column, you use zero as the previous value for G_L because G_L is 0 when there is no debt. The "Incremental %ΔV" is "Incremental ΔG_L" as a percentage of the previous column's firm value. For the "0.1" column, you use $10B as the previous value because firm value is $10B when there is no debt.]

Question 3. Computing Growth-Adjusted Costs of Borrowing

UGI wants to know if it can improve its unlevered firm value through a new line of marketable products for which future patents can assure constant long-term growth in cash payable to equity. The growth that UGI believes it can sustain needs a plowback ratio (PBR) 0.35. If growth adds to firm value, UGI will then use the G_L equation given by the CSM with growth to determine if leverage can further enhance its value beyond that computed for its non-growth leveraged situation. To use the CSM equation with growth, UGI must first estimate the levered growth rates (g_L) for its desired debt choices given its choice of a plowback ratio (PBR) of 0.35. Using the values and equations in Table 2, supply answers to the below questions.

Table 2. CSM (without growth) Values

PBR = plowback ratio used on CF_{BT} = **0.35**	**POR** = payout ratio = 1 – 0.35 = **0.65**
$\mathbf{T_C}$ = corporate tax rate = **30.00%**	$\mathbf{T_D}$ = personal tax rate on debt income = **15.00%**
α = 0.7823529411765	$\mathbf{r_U}$ **= 11.00%%**
$\mathbf{CF_{BT}}$ = perpetual before-tax cash flow generated by operating assets = **$1,654,135,338.34**	
RE = before-tax retained earnings = PBR(CF_{BT}) = 0.35($1,654,135,338.34) = **$578,947,368.42**	
C = before-tax cash to equity = (1 – PBR)(CF_{BT}) = 0.65($1,654,135,338.34) = **$1,075,187,969.92**	
I = Interest = $\frac{r_D D}{(1-T_D)}$ where D is D_{Miller} and I must be computed for each D choice. [Note. D is divided by $(1-T_D)$ because the below g_L equation uses an I value before personal taxes are considered and D includes personal taxes.]	
G = perpetuity cash flow (besides I) created with debt when $G_L \neq 0$. (Supplied for each D choice.)	
$\mathbf{g_U}$ = unlevered equity growth rate = $\frac{r_U(1-T_C)RE}{C} = \frac{0.11(1-0.3)\$529{,}323{,}308}{\$1{,}124{,}812{,}030}$ = **4.146153846%**	
$\mathbf{g_L}$ = levered equity growth rate = $\frac{r_L(1-T_C)RE}{C + G - \frac{I}{(1-T_C)}}$ (Computed for each D choice.)	
$\mathbf{r_{Ug}}$ = growth-adjusted unlevered equity rate of return = $r_U - g_U$ = 11% – 4.146154% = **6.853846%**	
$\mathbf{r_{Lg}}$ = growth-adjusted levered equity rate of return = $r_L - g_L$ (Computed for each D choice.)	
$\mathbf{V_U}$ (no growth) = $\frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U} = \frac{(1-0.05)(1-0.3)(1-0)\$1{,}654{,}135{,}338}{0.11-0.00}$ = **$10,000,000,000**.	

$$\mathbf{V_U}\text{ (growth)} = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U} = \frac{(1-0.05)(1-0.3)(1-0.35)\$1{,}654{,}135{,}338}{0.11 - 0.04146153846} = \mathbf{\$10{,}432{,}098{,}765.43}.$$

(a) From the V_U (no growth) and V_U (growth) values computed in Table 2, one can see that UGI increases its value through growth from undertaking its new line of products? Explain how this occurs?

(b) Fill in the blank cells in Exhibit 3.

(c) The CSM with growth offers the concept of "G" so as to express the gain to leverage (G_L value) as a perpetuity. G has been described as an enigmatic perpetuity cash flow because its value is difficult to determine even if G_L is estimated with confidence. This is because G can conceivably take on any number of perpetuity cash flow values depending on the value of r_{Lg}. Why is G positive in some of the cells in Exhibit 3, while negative in other cells? Explain.

(d) The CSM with growth develops the concept of "g_L" so as to express how a firm's growth in cash flows to equity changes when interest is paid out. Tests, using the CSM equation with growth, suggest that g_L will increase until too much debt causes financial distress problems at which point g_L will become negative and its computation breaks down. Based on this explanation what range of D values do you expect to contain the maximum amount of debt that firm will issue? Explain. (Hint: Use the fact that the equation for g_L in Table 2 suggests that a negative G value might lead to a negative g_L value when debt and thus I becomes large.)

Exhibit 3. Values for Growth Variables for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P = Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
$\mathbf{V_U}$ (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
D = P($\mathbf{V_U}$)	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
G	0.0544	0.1022	0.1407	0.1773	0.2188	–0.9366	–0.7763	–0.6134	–0.4654
$\mathbf{r_D}$	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%	8.18%	5.30%	10.28%
$\mathbf{r_L}$	11.12%	11.36%	11.84%	12.50%	13.28%	14.30%	15.50%	11.36%	18.44%
I	0.0621								1.1355
$\mathbf{g_L}$	4.330%								–7.382%
$\mathbf{r_{Lg}}$	6.790%								25.823%

[Note to Exhibit 3. As before, you should not round-off numbers until you have performed all computations; otherwise, errors can occur. For example, compute and use the more exact values for V_U, D, g_L, r_{Lg}, and G. Because you are not asked to compute G values, you should use the following G values that correspond to respective debt choice (or P values) from 0.1 to 0.9: \$54,381,590, \$102,153,829, \$140,719,080, \$177,341,522, \$218,817,110, –\$936,605,610, –\$776,316,593, –\$613,473,171, and –\$465,392,463. Values for the "0.1" and "0.9" columns are given to jump-start the computational process.]

Question 4. Computing CSM Values Using the CSM with Growth

Having estimated g_L values for each debt choice, UGI is now ready to determine its optimal debt choice using the G_L equation for the CSM with growth. This CSM equation is given by: $G_L = \left[1 - \frac{\alpha r_D}{r_{Lg}}\right] D - \left[1 - \frac{r_{Ug}}{r_{Lg}}\right] E_U$. Answer the below questions.

(a) Fill in the blank cells in Exhibit 4. For the r_{Lg} row, copy in the values computed previously. Identify and comment on the debt choice for UGI's maximum G_L and maximum V_L, the largest increase in its firm value (as given by the "%ΔV" row), and the optimal D/V_L.

(b) Explain the significance of the "Incremental ΔG_L" and "Incremental %ΔV" rows and what their first negative values indicate. Comment on how values for these two rows differ from Exhibit 2 when the CSM G_L equation was used without growth. Provide an explanation to account for the difference.

Exhibit 4. CSM (with growth) Values for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P = Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
$\mathbf{V_U}$ (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
D = P($\mathbf{V_U}$)	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
$\mathbf{r_D}$	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%	8.18%	5.30%	10.28%
r_{Lg}	6.790%								25.82%
$\mathbf{G_L}$	0.5326								–1.1985
$\mathbf{V_L}$	10.9647								9.2336
$\mathbf{E_L}$	9.9215								–0.1553
%ΔV	5.33%								–12.0%
Incremental ΔG_L	0.5326								0.4191
Incremental %ΔV	5.11%								4.75%
$\mathbf{D/V_L}$	0.0951								1.0168

[Note to Exhibit 4. As before, do not round-off numbers until you are ready to put them in the exhibit. Copy in the values for r_{Lg} computed in the previous question. Values for the "0.1" and "0.9" columns are given to jump-start the computational process. To compute values for any column, begin by computing G_L using the CSM equation with growth. Next, compute V_L given by $V_L = V_U + G_L$. E_L is given by $E_L = V_L - D$. The percentage change in firm value (%ΔV) is G_L as a percentage of V_U. The "Incremental ΔG_L" is the change in G_L from the previous column's G_L value. G_L for the first column uses zero as the previous value for G_L because G_L is 0 when there is no debt. The "Incremental %ΔV" is "Incremental ΔG_L" as a percentage of the previous column's firm value. For the "0.1" column, you use \$10.4321B (\$10,432,098,765 to be exact) as the previous firm value because there is no previous column value and firm value is \$10.4321B when there is no debt.]

Question 5. Computing and Comparing G_L Values

UGI wants to now compare all its G_L values computed from questions 1, 2, and 4 where it used the MM, Miller, and two CSM equations for the nine debt choices. Answer the below questions based on your previously computed G_L values.

(a) Fill in Exhibit 5 expressing all G_L values in billions of dollars and to four decimal places. In comparing the G_L values for all four equations, which equations are consistent with trade-off (or optimal) theory? Explain.

(b) Examine the G_L values for the first four debt choices and compare them in terms of dollar amounts. Based on your comparison of G_L values, would you conclude that a positive tax shield effect is the only explanation for a positive G_L? Explain.

(c) Which equation would you feel more comfortable with if you were a UGI manager charged with the capital structure decision? Explain.

Exhibit 5. Comparison of Values Given By Four G_L Equations

[Note. Express your answers in billions of dollars and to four decimal places.]

	P = Debt Choice (proportion of unlevered equity retired by debt)								
G_L Model	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
MM's G_L									
Miller's G_L									
CSM's G_L (no growth)									
CSM's G_L (growth)									

Question 6. Comparing Results for Different PBR Choices

While UGI's managers have chosen a PBR of 0.35, they are curious how other PBRs influence UGI's growth rates, firm value, and debt choice. Thus, they repeat their previous computations using other PBRs with a sample of their results given in Exhibit 6. Answer the below questions.

(a) Exhibit 6 does not provide results for a PBR less than 0.30. This is because UGI should not undertake growth as an unlevered firm unless it can attain an unleveraged growth rate (g_U) of at least 0.0330, which is the g_U associated with a PBR of 0.30. Illustrate why UGI's unlevered value (V_U) does not change with a PBR of 0.30 and how its V_U value falls if the PBR falls under 0.30. (Hint: You will have to use the r_U value and the two V_U equations in Table 1, and the CF_{BT} value and the RE, C, and g_U equations in Table 2.)

(b) From the values in Exhibit 6, is it possible to identify a PBR that maximizes firm value (V_L) for UGI? Does this PBR depend on UGI being able to sustain g_U? Explain.

(c) Do the PBRs and debt choices in Exhibit 6 indicate there is one plowback-payout choice and one debt-equity choice that together maximize firm value? Explain.

Exhibit 6. Computing Variable Values for Optimal PBRs and Debt Choices

[Note. Where applicable, express values in billions of dollars and to four decimals.]

PBR	**g_U**	**g_L**	**V_U**	**V_L**	**Debt Choice**

0.30	3.30%	7.59%	10.0000	12.3442	0.60
0.34	3.97%	7.15%	10.3223	12.6419	0.50
0.35	4.15%	7.54%	10.4321	12.9677	0.50
0.36	4.33%	7.95%	10.5567	13.3616	0.50
0.37	4.52%	8.38%	10.6981	13.8445	0.50
0.38	4.72%	7.10%	10.8588	13.1821	0.40
0.50	7.70%	7.70%	16.6667	16.6667	0.00

FINAL REMARKS

Student feedback when doing exercises involving CSM equations have been positive over the years from both upper level undergraduate finance students and graduate students. The following quote typifies students' feelings on the capital structure decision-making exercise:

> "*The CSM with growth model is most complete because it provides for more scenarios, thus being consistent with diversity of situations faced by managers in trying to determine the optimum leverage. The CSM framework can give due consideration to the growth that can be brought about if the company keeps aside some of its earning to fuel expansion.*"

The approval from students concerning the CSM applications has been received not only for upper level corporate finance courses taught in the classroom but also a graduate level course taught online.

Before presenting the exercise, we have found it advantageous to incorporate the exercise's formulas within one's lectures and handouts. By doing this, students will know what to expect and see that the equations can be readily used to generate answers to computational questions. However, since the exercise involves nine debt choices with repeated computations, we suggest that instructors use this exercise to also enhance a student's Excel spreadsheet skills.

To encourage student interaction and lower the amount of work, teachers can conduct the teaching exercise by assigning students to teams. See Hull, Roach and Weigand [2007] who offer some particulars when conducting a team exercise involving the collaborative aspects of peer learning. More can be expected when student works in teams. Within teams, there is more likelihood that at least one or two students will have advanced skills in Excel and PowerPoint and thus can more likely produce visual aids with tables, charts, and graphs to illustrate the optimal leverage choice and how G_L changes with the plowback choice or debt choice. Hull [2010] supplies two examples of graphs that could be done in Excel showing the influence of the plowback and debt choices. Alternately, student teams could be commissioned to find a desirable plowback and debt choice for a case study of an individual firm. A case study would challenge students to apply a CSM equation to a real firm of their choice (or a firm assigned by the instructor). Hull [2005] offers a procedure to unlever a firm so that a CSM equation can be used.

To accompany this paper's exercise, we have spreadsheets (identified by their tab names) that not only give detailed solutions to the questions asked in the previous section, but also solutions to simulatory applications of equation (4) when a variable is changed. These spreadsheets can be supplied electronically by requesting them. The spreadsheets strive to give sufficient details so that instructors and students will hopefully find them easy to use and understand. In particular, we highlight the solutions to the assigned questions and provide explanatory notes so the user can follow the computations that are being performed. We can note that any of these spreadsheets can be easily adapted by using costs of capital other than those generated by the CAPM. For example, Hull [2010] uses costs of capital influenced by the research of Hull [2005, 2007] and betas and debt ratings given by Pratt and Grabowski [2008]. The make-up of these spreadsheets is similar to those discussed by Hull [2008] except they focus on equation (4) instead of equation (3). The CSM applications found in these spreadsheets point out the potential influence of external factors (such as monetary and legislative policies) on the debt decision as well as market factors (such as signaling and agency considerations). For brevity's sake, the findings of these supplementary applications are not reported in this paper but are available on request.

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Appendix 1. Solutions to Question 1

(a) The below answers use the MM [1963] model that looks at value from a firm's viewpoint ignoring personal taxes.

(i) Noting that $C = (1-\text{PBR})(CF_{BT}) = (1-0)(\$1{,}654{,}135{,}338.34) = \$1{,}654{,}135{,}338.34$ and inserting this value and our other given values from Table 1 into $V_{UMM} = \frac{(1-T_{EMM})(1-T_{CMM})C}{r_U}$, we have: $V_{UMM} = \frac{(1-0)(1-0.3)(\$1{,}654{,}135{,}338.34)}{0.11} = \frac{0.7(\$1{,}654{,}135{,}338.34)}{0.11} = \frac{\$1{,}157{,}894{,}736.84}{0.11} = \mathbf{\$10{,}526{,}315{,}789}$.

(ii) Because UGI retires half of its unlevered equity value, its debt choice represented as P (the proportion of V_{UMM} retired) is 0.5. Expressing MM's debt as D_{MM}, we get: $D_{MM} = P(V_{UMM}) = 0.5(\$10{,}526{,}315{,}789) = \mathbf{\$5{,}263{,}157{,}895}$.

(iii) For MM, we use $G_L = T_C D$. Referring to G_L as G_{LMM} (the MM before-personal tax gain from leverage and D as D_{MM} (the MM before-personal tax value of debt), we have: $G_{LMM} = T_C D_{MM}$. Inserting in the Table 1 value of $T_C = 0.30$ and the value of $D_{MM} = \$5{,}263{,}157{,}895$ just computed in the previous problem, we have: $G_{LMM} = T_C D_{MM} = 0.3(\$5{,}263{,}157{,}895) = \mathbf{\$1{,}578{,}947{,}368}$.

(iv) $\frac{G_L}{V_U} = \frac{G_{LMM}}{V_{UMM}} = \frac{\$1{,}578{,}947{,}368}{\$10{,}526{,}315{,}789} = 0.1500$. Thus, the MM percentage increase in firm value caused by the debt issuance = $\mathbf{\%\Delta V_{MM} = 15.00\%}$.

(v) Noting MM's leveraged firm value (V_{LMM}) is its unleveraged firm value (V_{UMM}) plus its gain to leverage (G_{LMM}), MM's debt-to-firm value ratio is: $\mathbf{D_{MM}} / V_{LMM} = \frac{D_{MM}}{V_{UMM} + G_{LMM}} = \frac{\$5{,}263{,}157{,}895}{\$10{,}526{,}315{,}789 + \$1{,}578{,}947{,}368} = \mathbf{0.4348}$.

(vi) From an accounting standpoint, the perpetuity cash flow generated from operating cash flows can be viewed as the same as EBIT if the accounting term of EBIT represents actual cash flows. **Thus, we could have CF_{BT} = EBIT**. Because the firm is unlevered and pays no interest expense (I), EBT is the same as EBIT since EBIT – I = EBIT – 0 = EBT. **Thus, we could have CF_{BT} = EBT**. NI is EBT minus the corporate taxes paid, which in equation form is: NI = $(1-T_C)$EBT with NI < EBIT. **Thus, we could NOT have CF_{BT} = NI** if $T_C > 0$. Since there is no growth (e.g., no retained earnings), all of NI will be paid out as dividends and we have NI = DIV and thus DIV = NI < EBIT if $T_C > 0$. **Thus, we could NOT have CF_{BT} = DIV** if $T_C > 0$.

[Note. Because accounting statements treat items (such as depreciation) as a cash outflow, we know that only by chance will an accounting number equal an actual cash flow.]

(b) The below answers use the Miller [1977] model that looks at value from an investor's viewpoint that considers personal taxes.

(i) Recognizing that $V_{UMiller} = (1-T_{EMiller})V_{UMM}$, we have: $V_{UMiller} = (1-0.05)\$10{,}526{,}315{,}789 = \mathbf{\$10{,}000{,}000{,}000}$. We could also modify the previous MM equation for V_U with the Miller numbers to get the equation of $V_{UMiller} = \frac{(1-T_{EMiller})(1-T_{CMiller})(C)}{r_U}$. Inserting the values in Table 1 into this equation gives the same $10,000,000,000 answer.

(ii) We have: $D_{Miller} = (1-T_{EMiller})D_{MM} = (1-0.05)\$5{,}263{,}157{,}895 = \mathbf{\$5{,}000{,}000{,}000}$. We get the same answer using: $D_{Miller} = P(V_{UMiller}) = 0.5(\$10{,}000{,}000{,}000) = \$5{,}000{,}000{,}000$.

(iii) For Miller, we have: $G_L = [1-\alpha]D$. Referring to G_L as $G_{LMiller}$ (the Miller after-personal tax gain from leverage), α as α_{Miller} (the value for α using $T_{EMiller}$ and $T_{DMiller}$), and D as D_{Miller} (the Miller after-personal tax value of debt), we have: $G_{LMiller} = [1-\alpha_{Miller}]D_{Miller}$. Using $\alpha_{Miller} = \frac{(1-T_{EMiller})(1-T_C)}{(1-T_{DMiller})} = \frac{(1-0.05)(1-0.30)}{(1-0.15)} = 0.7823529411765$ and $D_{Miller} = \$5{,}000{,}000{,}000$ from part (ii), we have: $G_{LMiller} = [1-\alpha_{Miller}]D_{Miller} = [1-0.7823529411765]\$5{,}000{,}000{,}000 = \mathbf{\$1{,}088{,}235{,}294}$.

(iv) $\frac{G_L}{V_U} = \frac{G_{LMiller}}{V_{UMiller}} = \frac{\$1{,}088{,}235{,}294}{\$10{,}000{,}000{,}000} = 0.1088235294$. Thus, the Miller percentage increase in firm value caused by the debt issuance = $\mathbf{\%\Delta V_{Miller}}$ = 10.88235294% or about **10.88%**.

(v) Noting Miller's leveraged firm value ($V_{LMiller}$) is its unleveraged firm value ($V_{UMiller}$) plus its gain to leverage ($G_{LMiller}$), Miller's debt-to-firm value ratio is: $\mathbf{D_{Miller} / V_{LMiller}} = \frac{D_{Miller}}{V_{UMiller} + G_{LMiller}} = \frac{\$5{,}000{,}000{,}000}{\$10{,}000{,}000{,}000 + \$1{,}088{,}235{,}294} = \mathbf{0.4509}$.

(c) We begin by copying in the answers for the "0.5" column in Exhibit 1. These answers were previously computed in parts (a) and (b). We then follow the same computational procedure used in parts (a) and (b) to get the desired answers for the other columns of Exhibit 1. The answers for the empty cells of Exhibit 1 are given in bold print.

Given our assigned values, the MM and Miller equations both suggest that more debt is better. Thus, there is no optimal leverage ratio in the trade-off sense of G_L increasing with debt before decreasing. For a finite set of choices, the optimal leverage ratio is the one with the most debt, which would be the values in the last column for the "0.9" debt choice that retires 90 percent of the unlevered firm value. As seen in this column the debt-to-firm value ratios are 0.7087 for MM and 0.7526 for Miller. Both of these debt-to-firm value ratios greatly exceed the debt-to-firm value ratio for a typical firm, suggesting that the MM and Miller G_L equations fail to capture the negative leverage-related effects that govern the real world with its frictions that include bankruptcy and agency costs. In reality, most firms would only achieve such high leverage ratios unintentionally as when the value of their equity falls due to earnings problems. Furthermore, lenders would likely stop lending money to firms before such high leverage ratios could be reached (and even then lenders would charge an exorbitantly high interest rate that firms would not likely afford).

Exhibit 1. MM and Miller Values for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P** = **Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
$D_{MM} = P(V_{UMM})$	1.0526	**2.1053**	**3.1579**	**4.2105**	**5.2632**	**6.3158**	**7.3684**	**8.4211**	9.4737
G_{LMM}	0.3158	**0.6316**	**0.9474**	**1.2632**	**1.5789**	**1.8947**	**2.2105**	**2.5263**	2.8421
$\%\Delta V_{MM}$	3.00%	**6.00%**	**9.00%**	**12.00%**	**15.00%**	**18.00%**	**21.00%**	**24.00%**	27.00%
D_{MM} / V_{LMM}	0.0971	**0.1887**	**0.2752**	**0.3571**	**0.4348**	**0.5085**	**0.5785**	**0.6452**	0.7087
$D_{Miller} = P(V_{UMiller})$	1.0000	**2.0000**	**3.0000**	**4.0000**	**5.0000**	**6.0000**	**7.0000**	**8.0000**	9.0000
$G_{LMiller}$	0.2176	**0.4353**	**0.6529**	**0.8706**	**1.0882**	**1.3059**	**1.5235**	**1.7412**	1.9588
$\%\Delta V_{Miller}$	2.18%	**4.35%**	**6.53%**	**8.71%**	**10.88%**	**13.06%**	**15.24%**	**17.41%**	19.59%
$D_{MM} / V_{LMiller}$	0.0979	**0.1917**	**0.2816**	**0.3680**	**0.4509**	**0.5307**	**0.6075**	**0.6814**	0.7526

Appendix 2. Solutions to Question 2

(a) We fill in all empty cells in Exhibit 2 and show that the maximum G_L is **\$1.3331B**. This G_L occurs in the "0.5" column, which is the column that represents a debt choice (or P value) of **0.5**. This column also contains the greatest V_L of **\$11.3331B** and the maximum percentage change in firm value (%ΔV) of **13.33%**. Moving down the "0.5" column, we find that the optimal debt-to-firm value ratio (D / V_L) is **0.4412**. The ratio of 0.4412 means UGI should finance its projects with \$44.12 of debt for every \$100 of total financing from both debt and equity.

(b) As seen in Exhibit 2, if UGI chooses more than \$5.0000B in debt, the incremental change in the gain to leverage ("Incremental ΔG_L") and the incremental percentage change in firm value ("Incremental %ΔV") both become negative. For example, for the \$6.0000B debt value associated with the "0.6" column, we see respective values of **–\$0.0503B** and **–0.44%**. These are the first negative values that occur for these two rows. Thus, if we go past the \$5.0000B debt value, UGI's firm value will begin falling. Subsequent values for these two rows in the next three columns are increasingly negative indicating even greater leverage-related costs as the debt choice increases.

Below we illustrate the computations in Exhibit 2 for the "0.5" column, which is the column where UGI maximizes its value by issuing \$5.0000B in debt to retire one-half its equity. Using the CSM GL equation and inserting the previous values (including α = 0.7823529411765 from Table 1), we have:

$$\mathbf{G_L}\left(\mathbf{0.5\ column}\right) = \left[1-\frac{\alpha r_D}{r_L}\right]D - \left[1-\frac{r_U}{r_L}\right]V_U = \left[1-\frac{0.7823529411765(0.0662)}{0.1328}\right]\$5B - \left[1-\frac{0.11}{0.1328}\right]\$10B \Rightarrow$$

$$\mathbf{G_L}\left(\mathbf{0.5\ column}\right) = \$3{,}050{,}008{,}859 - \$1{,}716{,}867{,}470 \Rightarrow \mathbf{G_L = \$1{,}333{,}141{,}389} \text{ or about } \mathbf{\$1.3331B}.$$

We can compute the value of the levered firm for the 0.5 debt choice as:

$$\mathbf{V_L}\left(\mathbf{0.5\ column}\right) = V_U + G_L\left(0.5\text{ column}\right) = \$10{,}000{,}000{,}000 + \$1{,}333{,}141{,}389 \Rightarrow$$

$$\mathbf{V_L = \$11{,}333{,}141{,}389} \text{ or about } \mathbf{\$11.3331B}.$$

The value of the levered equity for the 0.5 debt choice is:

$$\mathbf{E_L}\left(\mathbf{0.5\ column}\right) = V_L\left(0.5\text{ column}\right) - D\left(0.5\text{ column}\right) = \$11{,}333{,}141{,}389 - \$5B \Rightarrow$$

$$\mathbf{E_L = \$6{,}333{,}141{,}389} \text{ or about } \mathbf{\$6.3331B}.$$

The percentage change in firm value for the 0.5 debt choice is:

$$\frac{\mathbf{G_L}\left(\mathbf{0.5\ column}\right)}{\mathbf{V_U}} = \frac{\$1.333141B}{\$10.000B} = 0.1333141 \Rightarrow \%\Delta V_U\left(0.5\text{ column}\right) = \mathbf{\%\Delta V} = \text{about } \mathbf{13.33\%}.$$

The incremental change in G_L for the 0.5 debt choice is:

$$\mathbf{Incremental\ \Delta G_L}\left(\mathbf{0.5\ column}\right) = G_L\left(0.5\text{ column}\right) - G_L\left(0.4\text{ column}\right) = \$1.333141B - \$1.292876B \Rightarrow$$

$$\mathbf{Incremental\ \Delta G_L = \$40{,}266{,}095} \text{ or about } \mathbf{\$0.0403B.}$$

The incremental percentage change in firm value for the 0.5 debt choice is:

$$\mathbf{Incremental\ \%\Delta V}\left(\mathbf{0.5\ column}\right) = \frac{\text{Incremental } \Delta G_L\left(0.5\text{ column}\right)}{V_L\left(0.4\text{ column}\right)} = \frac{\$0.0403B}{\$11.2929B} \Rightarrow$$

$$\mathbf{Incremental\ \%\Delta V = 0.00356562} \text{ or about } \mathbf{0.36\%}.$$

Finally, we compute the debt-to-firm value ratio for 0.5 debt choice. Doing this, we get:

$$\frac{\mathbf{D}\left(\mathbf{0.5\ column}\right)}{\mathbf{V_L}\left(\mathbf{0.5\ column}\right)} = \frac{\$5.000B}{\$11.3331B} = 0.4411839 \Rightarrow \mathbf{D/V_L} = \text{about } \mathbf{0.4412}.$$

Exhibit 2. CSM (without growth) Values for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P** = **Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
V_U (or E_U)	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
D = P(V_U)	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000	9.0000
$\mathbf{r_D}$	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%	8.18%	5.30%	10.28%
$\mathbf{r_L}$	11.12%	11.36%	11.84%	12.50%	13.28%	14.30%	15.50%	11.36%	18.44%

G_L	0.5361	0.9531	1.1804	1.2929	**1.3331**	1.2829	1.2066	0.9531	1.0400
V_L	10.5361	10.9531	11.1804	11.2929	**11.3331**	11.2829	11.2066	10.9531	11.0400
E_L	9.5361	8.9531	8.1804	7.2929	6.3331	5.2829	4.2066	8.9531	2.0400
%ΔV	5.36%	9.53%	11.80%	12.93%	**13.33%**	12.83%	12.07%	9.53%	10.40%
Incremental ΔG_L	0.5361	0. 4170	0.2274	0.1124	0.0403	**–0.0503**	–0.0763	0. 4170	–0.0877
Incremental %ΔV	5.36%	3.96%	2.08%	1.01%	0.36%	**–0.44%**	–0.68%	3.96%	–0.79%
D/V_L	0.0949	0.1826	0.2683	0.3542	**0.4412**	0.5318	0.6246	0.1826	0.8152

Appendix 3. Solutions to Question 3

(a) As seen in Table 2, growth increases UGI's unlevered value from V_U (no-growth) of \$10,000,000,000 to V_U (growth) of \$10,432,098,765.43. UGI increases its value by being able to sustain its before-tax plowback ratio (PBR) of 0.35 as given in Table 2. One obvious and simple answer as to how a PBR of 0.35 increases value would be that the earnings plowed back are more valuable than what investors could achieve by having it paid out as cash. In other words, the value of the cash flows from retained earnings is greater than the value obtained from just paying it out.

[Note. A more complete answer is not obvious unless one understands the exact valuation impact that a sustainable PBR of 0.35 achieves with all expenses considered. In the developing the CSM with growth, Hull [2010] argues that the costs from retaining earnings for growth are determined by the effective corporate tax rate (T_C). These costs can be described as the "first corporate taxes paid" when retained earnings are invested. The "second corporate taxes paid" comes later when the cash flows created from the retained earnings are also taxed at the corporate level. To keep non-growth firm value for an unlevered firm equal to its growth value, Hull [2010] posits that the minimum PBR = T_C and the minimum g_U = (minimum PBR)(r_U) = $T_C(r_U)$. This is shown as follows. With $T_C = 0.3$ and $r_U = 0.11$, we get: minimum PBR = T_C = 0.30 and minimum g_U = (minimum PBR)(r_U) = $T_C(r_U) = 0.3(0.11) = 0.0330$. Using the minimum PBR of 0.30 for PBR and the minimum g_U of 0.0330 for g_U in the V_U (growth) equation given in Table 2, we have:

$$V_U\text{ (growth)} = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U} = \frac{(1-0.05)(1-0.3)(1-0.3)\$1{,}654{,}135{,}338}{0.11 - 0.0330} =$$

$$\$\mathbf{10{,}000{,}000{,}000}.$$

This value of \$10B is the same value found in Table 2 for V_U with no growth. Keep in mind, this equation assumes the use of internal equity. If the firm used external equity, Hull [2010] argues that a firm is able to avoid the double corporate taxation from the use of retained earnings such that the minimum g_U could be much less than 0.0330 because the issuance costs associated with external equity are, on average, only about 1/5 of the costs associated with paying an extra corporate tax.]

(b) We fill in all blank cells in Exhibit 3 by using values supplied in Table 2 for T_C, T_D, RE, and C; equations given in Table 2 for I, g_L, and r_{Lg}; and values supplied in Exhibit 3 for P, V_U, G, r_D, and r_L (where precise values for G are given in the note to Exhibit 3). As explained in Table 2, the interest (I) formula computes I on a before-tax basis because the levered equity growth rate (g_L) formula uses this I value. We illustrate the computations for I, g_L, and r_{Lg} when the debt choice is 0.5. All other computations for these three variables are computed in the same fashion for each debt choice (or P value).

Precise value for **D** = P(V_U) = 0.5($10,432,098,765.43) = **$5,216,049,382.72** or about **$5.2160B**.

$$\mathbf{I} = \text{Interest} = \frac{r_D D}{(1-T_D)} = \frac{0.0662(5,216,049,382.72)}{(1-0.15)} = \mathbf{\$406,238,198.98} \text{ or about } \mathbf{\$0.4062B}.$$

$$\mathbf{g_L} = \frac{r_L\,(1-T_C)\,RE}{C + G - \frac{I}{(1-T_C)}} \Rightarrow$$

$$\mathbf{g_L} = \frac{0.1328(1-0.3)\$578,947,368.42}{\$1,075,187,969.92 + \$218,817,110 - \frac{\$406,238,198.98}{(1-0.03)}} = \mathbf{0.075412081} \text{ or about } \mathbf{7.541\%}.$$

$$r_{\mathbf{Lg}} = r_L - g_L = 0.1328 - 0.075412081 = \mathbf{0.05738792} \text{ or about } \mathbf{5.739\%}.$$

(c) G is positive in its first five cells in Exhibit 3 (and negative in its last four cells). A simple reason is the following. Because G is a perpetuity stemming from G_L, positive values in the first five cells for G can only result if G_L is positive for these D values. The positive values for G_L in these five cells will be seen in the next problem when we compute G_L values as they will be positive for D values up to $5.2160B. Similarly, negative values for G_L give negative G values.

(d) If g_L does become negative when too much debt is issued causing $G_L < 0$ to hold, then g_L will reach a point before it becomes negative where V_L will be maximized. Based on the G values in Exhibit 3, we would expect this maximum V_L to occur before the debt choice of P = 0.60 is achieved because this is when G first becomes negative and thus g_L would be expected to become negative. Consequently, one would expect the maximum D to be somewhere from zero debt to $5.0000B in debt. The solutions to the next question will confirm that this expectation holds.

Exhibit 3. Values for Growth Variables for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P** = **Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
V_U (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
D = P(**V_U**)	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
G	0.0544	0.1022	0.1407	0.1773	0.2188	–0.9366	–0.7763	–0.6134	–0.4654
r_D	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%	8.18%	9.14%	10.28%
r_L	11.12%	11.36%	11.84%	12.50%	13.28%	14.30%	15.50%	16.88%	18.44%
I	0.0621	**0.1301**	**0.2062**	**0.2955**	**0.4062**	**0.5405**	**0.7028**	**0.8974**	1.1355
g_L	4.330%	**4.643%**	**5.208%**	**6.101%**	**7.541%**	**–9.147%**	**–8.909%**	**–8.340%**	–7.382%
r_{Lg}	6.790%	**6.717%**	**6.632%**	**6.399%**	**5.739%**	**23.447%**	**24.409%**	**25.230%**	25.823%

Appendix 4. Solutions to Question 4

(a) We fill in all empty cells in Exhibit 4 and show that the maximum G_L is **$2.5356B**.

This G_L value occurs in the "0.5" column, which is the column that represents a debt choice (or P value) of **0.5**. This column also contains the greatest V_L value of **\$12.9677B** and the maximum percentage increase in firm value (%ΔV**)** of **25.36%**. Moving down the "0.5" column, we find that the optimal debt-to-firm value ratio (D/V_L) is **0.4022**.

(b) As seen in Exhibit 4, if UGI chooses more than \$5.2160B in debt, the incremental change in the gain to leverage and the incremental percentage change in firm value both become negative. For example, looking at the "Incremental ΔG_L" and "Incremental %ΔV" rows for the \$6.2593B debt value associated with the "0.6" column, we see respective values of –**\$5.1920B** and **–40.04%**. These are the first negative values that occur for these two rows. Thus, if we go past the \$5.2160B debt value, UGI's firm value will be lowered. Unlike the results in Exhibit 2 for the non-growth situation, subsequent values for these two rows in the next three columns are not increasingly negative and thus do not appear to indicate greater financial distress costs if managers choose greater debt levels. One reason for the difference can be seen from equation for g_L where a break-down occurs when G_L becomes negative. This is because a negative G_L value renders a negative G, which causes g_L to become negative. While the Dividend Valuation Model becomes non-functional when growth rates become too large, the CSM becomes non-functional when g_L becomes negative.

Exhibit 4. CSM (with growth) Values for Debt Choices

[Note. Where applicable, values are expressed in billions of dollars and to four decimal places.]

Variables	**P** = **Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
$\mathbf{V_U}$ (growth)	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321	10.4321
D = P($\mathbf{V_U}$)	1.0432	2.0864	3.1296	4.1728	5.2160	6.2593	7.3025	8.3457	9.3889
$\mathbf{r_D}$	5.06%	5.30%	5.60%	6.02%	6.62%	7.34%	8.18%	5.30%	10.28%
$r_{\mathbf{Lg}}$	6.790%	6.717%	6.632%	6.399%	5.739%	23.45%	24.41%	25.23%	25.82%
$\mathbf{G_L}$	0.5326	1.0114	1.4110	1.8429	**2.5356**	–2.6564	–2.1150	–1.6176	–1.1985
$\mathbf{V_L}$	10.9647	11.4435	11.8431	12.2750	**12.9677**	7.7757	8.3171	8.8145	9.2336
$\mathbf{E_L}$	9.9215	9.3571	8.7135	8.1022	7.7517	1.5165	1.0146	0.4688	–0.1553
%ΔV	5.33%	10.11%	14.11%	18.43%	**25.36%**	–26.6%	–21.2%	–16.2%	–12.0%
Incremental $\mathbf{\Delta G_L}$	0.5326	0.4788	0.3996	0.4320	0.6927	**–5.1920**	0.5414	0.4973	0.4191
Incremental %ΔV	5.11%	4.37%	3.49%	3.65%	5.64%	**–40.04%**	6.96%	5.98%	4.75%
$\mathbf{D/V_L}$	0.0951	0.1823	0.2643	0.3399	**0.4022**	0.8050	0.8780	0.9468	1.0168

Below we illustrate the computations in Exhibit 4 for the "0.5" column, which is the column where UGI maximizes its value by issuing \$5.2160B in debt to retire one-half its equity. Using the CSM equation for G_L with growth and inserting the previous values (including α from Table 1 and making sure other values are expressed so as to avoid rounding off errors), we have:

$$\mathbf{G_L}\left(\mathbf{0.5\ column}\right) = \left[1-\frac{\alpha r_D}{r_{Lg}}\right]D - \left[1-\frac{r_{Ug}}{r_{Lg}}\right]E_U \Rightarrow$$

$$\mathbf{G_L}\left(\mathbf{0.5\ column}\right) =$$

$$\left[1-\frac{0.7823529411765(0.0662)}{0.057387919}\right]\$5{,}216{,}049{,}383 - \left[1-\frac{0.068538462}{0.057387919}\right]\$10{,}432{,}098{,}765 \Rightarrow$$

$$\mathbf{G_L}\left(\mathbf{0.5\ column}\right) = \$508{,}640{,}455 - \$2{,}026{,}969{,}490 \Rightarrow \mathbf{G_L = \$2{,}535{,}609{,}945} \text{ or about } \mathbf{\$2.5356B}.$$

We can compute the value of the levered firm for the $5.2160Bdebt value (which is the 0.5 debt choice) as:

$$\mathbf{V_L}\left(\mathbf{0.5\ column}\right) = V_U + G_L\left(0.5\ \text{column}\right) = \$10{,}432{,}098{,}765 + \$2{,}535{,}609{,}945 \Rightarrow$$

$$\mathbf{V_L = \$12{,}967{,}708{,}710} \text{ or about } \mathbf{\$12.9677B}.$$

The value of the levered equity for the 0.5 debt choice is:

$$\mathbf{E_L}\left(\mathbf{0.5\ column}\right) = V_L\left(0.5\ \text{column}\right) - D\left(0.5\ \text{column}\right) = \$12{,}967{,}708{,}710 - \$5{,}216{,}049{,}383 \Rightarrow$$

$$\mathbf{E_L = \$7{,}751{,}659{,}327} \text{ or about } \mathbf{\$7.7517B}.$$

The percentage change in firm value for the 0.5 debt choice is:

$$\mathbf{\%\Delta V_U}\left(\mathbf{0.5\ column}\right) = \frac{G_L\left(0.5\ \text{column}\right)}{V_U} = \frac{\$2{,}535{,}609{,}945}{\$10{,}432{,}098{,}765} = \mathbf{0.25356099} \text{ or about } \mathbf{25.36\%}.$$

The incremental change in G_L for the 0.5 debt choice is:

$$\textbf{Incremental } \mathbf{\Delta G_L}\left(\mathbf{0.5\ column}\right) = G_L\left(0.5\ \text{column}\right) - G_L\left(0.4\ \text{column}\right) \Rightarrow$$

$$\textbf{Incremental } \mathbf{\Delta G_L} = \$2{,}535{,}609{,}945 - \$1{,}842{,}945{,}166 = \mathbf{\$692{,}664{,}779} \text{ or about } \mathbf{\$0.6927B}.$$

The incremental percentage change in firm value for the 0.5 debt choice is:

$$\textbf{Incremental } \mathbf{\%\Delta V}\left(\mathbf{0.5\ column}\right) = \frac{\text{Incremental } \Delta G_L\left(0.5\ \text{column}\right)}{V_L\left(0.4\ \text{column}\right)} = \frac{\$692{,}664{,}779}{\$12{,}967{,}708{,}710} \Rightarrow$$

$$\textbf{Incremental } \mathbf{\%\Delta V} = \mathbf{0.05642870} \text{ or about } 5.64\%.$$

Finally, we compute the debt-to-firm value ratio for 0.5 debt choice. Doing this, we get:

$$\frac{\mathbf{D}\left(\mathbf{0.5\ column}\right)}{\mathbf{V_L}\left(\mathbf{0.5\ column}\right)} = \frac{\$5{,}216{,}049{,}383}{\$12{,}967{,}708{,}710} \Rightarrow \mathbf{D/V_L = 0.4022}.$$

Appendix 5. Solutions to Question 5

(a) In Exhibit 5, we give the MM, Miller and CSM G_L values computed previously. In comparing the G_L values, we see that the MM equation gives increasing values for G_L implying that more debt is better. The Miller model also gives increasing G_L values albeit the values are smaller due to the greater personal tax disadvantage of debt compared to equity (because $T_D > T_E$ in our exercise). Thus, at least for our personal tax values, the general conclusion for the Miller equation is like the MM equation: the more debt the better. In analyzing Exhibit 5, we see that CSM equation with no growth renders numbers consistent with trade-off theory, which predicts rising G_L values until the optimal debt level is reached and at which point G_L values decline. This rise and decline in G_L occurs because a CSM equation allows the costs of capital for debt and equity to increase (as dictated by an increase in financial risk that concomitantly leads to greater systematic risk). When we use the CSM equation with growth, we get results similar to the CSM without growth with these noticeable differences. *First*, the G_L values using the CSM with growth tend to be much more positive until they become negative. *Second*, the negative values for the CSM with growth do not worsen with more debt (the reasons discussed previously when we noted the break-down in computing g_L when G_L becomes negative).

(b) One can notice that the first four debt choices using both CSM equations render greater positive G_L values compared to either MM or Miller. The greater positive values can be attributed not only to the positive tax shield effect but to the fact r_D is less than r_L in the 1st component of the CSM non-growth equation (and less than r_{Lg} in the 1^{st} component of the CSM growth equation for lower debt choices). This can be seen by setting $\alpha = 1$ so as to make the net tax effect zero. Even for this situation, the 1^{st} components of CSM equations can still generate positive values. Hull [2007] suggests that these positive values can be attributed simply to the way ownership claims are packaged and sold to shield the firm from agency costs.

(c) As a financial manager, you want an equation capturing all of the leverage-related effects. Whereas the MM equation is very simple and thus commonly referred to when discussing the advantage of debt, you might feel more comfortable with an equation (like a CSM equation) that is capable of capturing (i) the positive effects that go beyond a tax shield effect and (ii) the negative effects of debt. After the optimal debt level is reached, G_L values using the CSM equation with no growth decline because the negativity of its 2^{nd} component begins to dominate; for the CSM equation with growth, negativity occurs for each subsequent debt choice once G_L becomes negative even though the decline does not become increasing worse (due to the CSM becoming non-functional when g_L becomes negative). For this paper's exercise, relying on the MM and Miller equations causes a firm to issue too much debt. In practice, a firm might make its debt choice based on a desired bond rating. This debt choice is much likely to be more consistent with a debt choice recommended by the CSM equations for G_L than by the MM and Miller equations for G_L.

Exhibit 5. Comparison of Values Given By Four G_L Equations

[Note. All values are expressed in billions of dollars to four decimals.]

G_L Model	**P = Debt Choice** (proportion of unlevered equity retired by debt)								
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
MM's G_L	0.3158	0.6316	0.9474	1.2632	1.5789	1.8947	2.2105	2.5263	2.8421
Miller's G_L	0.2176	0.4353	0.6529	0.8706	1.0882	1.3059	1.5235	1.7412	1.9588
CSM's G_L (no growth)	0.5361	0.9531	1.1804	1.2929	1.3331	1.2829	1.2066	1.1276	1.0400
CSM's G_L (growth)	0.5326	1.0114	1.4110	1.8429	2.5356	–2.6564	–2.1150	–1.6176	–1.1985

Appendix 6. Solutions to Question 6

(a) Below we show that UGI would not want to choose a PBR less than 0.30 because it would lower its unlevered firm value. Let us choose PBR = 0.25 (even though we could use any PBR under 0.30). To compute V_U, we first compute g_U. For PBR = 0.25, we have: RE = PBR(CF_{BT}) = 0.25($1,654,135,338) = $413,533,835 and C = (1 − PBR)(CF_{BT}) = (1 − 0.25)($1,654,135,338) = $1,240,601,504. Thus, $g_U = \frac{r_U(1-T_C)RE}{C} = \frac{0.11(1-0.3)\$413{,}533{,}835}{\$1{,}240{,}601{,}504} =$ **0.02566666667** or about **2.5667%**. Because POR = 1 − PBR = 1 − 0.25 = 0.75 and given that it can be shown that RE/C = PBR/POR, we have: $g_U = r_U(1-T_C)(PBR/POR) = 0.11(1-0.3)(0.25/0.75) =$ **0.02566666667**. Similarly, For PBR = 0.30, we have: $g_U = r_U(1-T_C)(PBR/POR) = 0.11(1-0.3)(0.3/0.7) =$ **0.0330** or **3.30%**. We now compute V_U values for PBR = 0 (no growth), 0.25, and 0.30. We have:

$$\mathbf{V_U}\text{ (no growth)} = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U} = \frac{(1-0.05)(1-0.3)(1-0)\$1{,}654{,}135{,}338}{0.11-0.00} = \mathbf{\$10{,}000{,}000{,}000}.$$

$$\mathbf{V_U}\,(\text{growth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U} = \frac{(1-0.05)(1-0.3)(1-0.25)\$1,654,135,338}{0.11\;-\;0.02566666667} = \mathbf{\$9,782,608,696}.$$

$$\mathbf{V_U}\,(\text{growth}) = \frac{(1-T_E)(1-T_C)(1-PBR)CF_{BT}}{r_U - g_U} = \frac{(1-0.05)(1-0.3)(1-0.3)\$1,654,135,338}{0.11\;-\;0.0330} = \$\mathbf{10,000,000,000}.$$

Thus, we see that a PBR ratio under 0.30 decreases unleveraged firm value, while a PBR of 0.30 maintains the same unleveraged firm value as the no growth situation where PBR = 0. We can use other PBR values less than 0.30 to illustrate why a firm seeking to grow through internal equity cannot choose a PBR under 0.30. For example, PBRs of 0.01, 0.15, and 0.29 give V_U (growth) values of $9,970,498,474, $9,697,986,577, and $9,942,800,789.

(b) From Exhibit 6, we see that it is possible to identify a PBR that maximizes V_L. For example, a PBR of 0.50 would generate the highest V_U value of $16.6667B. For this PBR, you would issue no debt. The question a manager must face is whether a growth rate of 7.70% is sustainable. For the short-term such a growth rate can be sustained; however, for our perpetuity model (where the growth rate extends for a long period of time), it is quite possible that 7.70% as well as the larger g_U values given in Exhibit 6 are not sustainable by most companies. For example, the average growth rate across countries is typically given at a percentage under 4%. If that is the case, then most of the PBR choices in the 1st column of Exhibit 6 are not feasible. UGI has assumed that a growth rate of 4.15% is sustainable given its choice of PBR = 0.35. With this PBR, UGI would maximize firm value with a debt choice of 0.50, thus retiring one-half of its unlevered firm value (V_U). By issuing this much debt, UGI actually succeeds in having its remaining equity owner's cash flow grow by a larger rate as indicated by its g_L of 7.54%.

(c) Given a firm's constraints over how much it can grow and assuming no rigid constraints on debt borrowing, values in Exhibit 6 indicate there is one plowback-payout choice and one debt-equity choice that together maximize firm value. Since a greater PBR increases firm value, managers must determine what maximum PBR can be sustained. Once this PBR is chosen, then managers must determine which debt choice combines with this PBR choice to maximize firm value. For example, PBRs of 0.30, 0.35, 0.38, and 0.50 all generate different debt choices. Thus, the choice of PBR determines the debt choice and we can say that the plowback-payout choice determines the debt-equity choice. On the other hand, suppose managers first pick a target debt choice and then go about determining a PBR choice that maximizes V_L. Given this debt choice, a manager could try all feasible PBRs to find out which one generates the maximum V_L.