Interest Rate Swap Pricing: A Classroom Primer

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ABSTRACT

In this paper I present an introductory lesson on interest rate swaps and two models for interest rate swap valuation. I begin by outlining important concepts of the interest rate swap market. I show that a swap can be valued based on simple present value techniques and using implied forward rates, with the same result. I provide a basic model for computing an at-market swap rate. In all three stages, I give swap valuation examples that can be applied in an undergraduate or graduate finance curriculum.

INTRODUCTION

The interest rate swap market is the largest and fastest growing derivative market. Interest rate swaps are important tools for hedgers, speculators, and investors. According to a survey performed by the Bank for International Settlements, the outstanding volume of interest rate swaps was \$309.6 trillion as of the end of 2007.¹ This was an increase of 34.78% from year-end 2006. Given the size and growth of the interest rate swap market, it is a topic that cannot be ignored in undergraduate and graduate curriculums.

Understanding interest rate swap pricing is critical to the understanding of the mechanics of interest rate swaps. Some of the interest rate debacles of the past were due to a misunderstanding of the correct valuation of interest rate swaps.² For these reasons, it is important to introduce students to basic pricing models that capture the key aspects of the interest rate swap market. This paper presents two relatively simple swap pricing models that emphasize the important characteristics of interest rate swaps.

The first model is based on basic present value techniques. It serves as an introduction to interest rate swap valuation. The second model is more complex and involves implied forward rates. It can be used to value swaps with more complex structures. I also explain the calculation of an at-market swap rate. This is helpful in completing students' understanding of the interest rate swap market.

¹ http://www.bis.org/statistics/derstats.htm

 $^{^{2}}$ A good example of this is the Bankers Trust and Procter and Gamble interest rate swap transaction that was unwound in 1994. See Smith (1997).

This paper is organized as follows. First, I describe the basic structure of an interest rate swap. Then a simple methodology for valuing a swap based on present value techniques is presented. I then introduce implied forward rates and develop a more accurate valuation technique. Finally, I present an example of how to compute an at-market swap rate.

VANILLA INTEREST RATE SWAPS

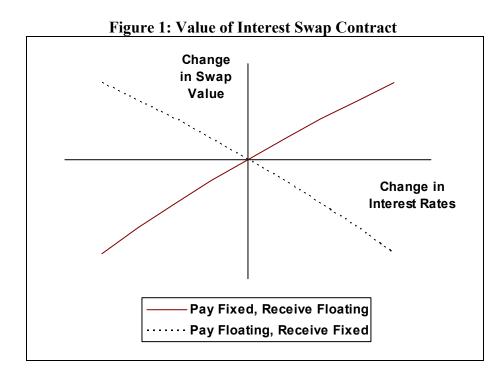
A *plain vanilla* interest rate swap is a contract under which two counterparties, a floating-rate payer and a fixed-rate payer, agree to exchange net payments at a series of future points of time. A notional principal amount is used to calculate the amount of the net payments. An interest rate swap can have a maturity or *tenor* in excess of 30 years.

The fixed swap rate is set on the pricing date of the swap. The floating rate is established for the first payment based on market levels on the pricing date and reset periodically based on market levels. The floating rate on a plain vanilla swap is based on 3-month LIBOR. Interest rates are typically reset quarterly with net payments made semi-annually.

An interest rate swap can be structured with countless variations to this structure. Example floating rate benchmarks include the commercial paper rate, the U.S. Treasury-bill rate, 1-month LIBOR, and the SIFMA index. Payments can be calculated and made monthly, quarterly or at some other desired periodicity. A spread to the floating rate can also be added based on market conditions or the needs of the swap initiator.

In an interest rate swap, the fixed rate payer is said to be the buyer and the floating rate payer is said to be the seller; however, from an economic or hedging standpoint the opposite is true. The value of an interest rate swap changes with changes in interest rates. Figure 1 shows the change in the value of a swap relative to changes in interest rates. Although these appear to be straight lines, they represent the actual change in the value of a hypothetical swap given a change in interest rates.

For the fixed rate payer (floating rate receiver) the value of the swap increases when interest rates increase. This means that a portfolio manager that is long bonds can effectively hedge the value of this position by entering into a swap that requires the manager to pay fixed and receive floating. For the floating rate payer (fixed rate receiver) the value of the swap increases when interest rates decrease.



THE ZERO-COUPON YIELD CURVE

The zero-coupon yield, or spot rate of interest, is the yield-to-maturity on a bond or investment with only one cash flow occurring on a specific date or maturity. U.S. Treasury Bills and U.S. Treasury STRIPS are examples of default-free zero-coupon securities. U.S. Treasury STRIPS are often used to construct a representative risk-free taxable zero-coupon curve. STRIPS, an acronym for Separately Traded Registered Interest and Principal Securities, represent zero-coupon bonds that are created by stripping full-coupon U.S. Treasury bonds. One problem with relying on actively traded zero-coupon bonds is that they may not be traded at desired maturities and yields may have to be interpolated between maturities. For this reason, zero-coupon yields are often calculated from full coupon rates. A yield curve constructed from default-free zero-coupon bonds is known as the term structure of interest rates. Swap pricing is based on the zero-coupon yield curve.

It is often the case that coupon bond prices are available, but zero-coupon bond prices are not available. In these circumstances, it is possible to estimate zero-coupon bond prices from full-coupon bond prices using a procedure known as bootstrapping.³ The yields associated with these estimated zero-coupon bond prices are known as implied zero-coupon yields. Implied zero-coupon yields are the set of discount rates implied in the par yield curve that equates the cash flows of a full-coupon bearing bond to those of a set of zero-coupon

³ In practice, swap yield curves are constructed using a combination of money rates, Eurodollar futures, and market swap rates. For a more detailed discussion see Cusatis and Thomas (2006) and Young (1997).

bonds. The theoretical implied zero-coupon yield adjusts the full-coupon rate for the loss or gain associated with the periodic reinvestment of interest payments.

To illustrate this concept, assume that a non-callable, full-coupon bond that matures in n years, makes a payment of C/2 in each time period t, t=1,2,3,...2n. At maturity, the fullcoupon bond also pays the \$100 face value. If the is currently selling at par, the bond price is calculated as

$$100 = \sum_{t=1}^{2n} \frac{\frac{C}{2}}{\left(1 + \frac{y}{2}\right)^t} + \frac{100}{\left(1 + \frac{y}{2}\right)^{2n}}$$

To calculate implied zero-coupon yields using the bootstrapping procedure, we replace the yield to maturity, y, with the appropriate zero-coupon yield in each period. A set of zero-coupon yields, r_t , exists that, when used as discount rates, will result in the current price of the bond. We begin with the zero-coupon yield for the shortest period of time and calculate the implied zero-coupon yields for subsequent periods. The zero-coupon yield for the shortest time period can be observed in the market. The bootstrapping procedure to derive an implied zero-coupon curve is illustrated by the following example.

Example: Table 1 presents data for ten coupon bonds selling at par. Bond one, which matures in one period has an annualized coupon rate of 2.00%. The implied one-period zero-

coupon yield, $\frac{r_1}{2}$, is equal to 1.00%, the semiannual market yield on bond one.

| Bond | Maturity (t) | Coupon Rate |
|------|--------------|-------------|
| 1 | 1.0 | 2.00% |
| 2 | 2.0 | 2.25% |
| 3 | 3.0 | 2.50% |
| 4 | 4.0 | 3.00% |
| 5 | 5.0 | 3.40% |
| 6 | 6.0 | 3.70% |
| 7 | 7.0 | 4.10% |
| 8 | 8.0 | 4.45% |
| 9 | 9.0 | 4.70% |
| 10 | 10.0 | 5.00% |

Table 1: Annualized Yields on Coupon Bonds

The implied zero-coupon yield for period 2, $\frac{r_2}{2}$, is calculated by solving the following equation:

$$100 = \frac{1.125}{(1.010)} + \frac{101.125}{(1 + \frac{r_2}{2})^2}.$$

Solving for $\frac{r_2}{2}$, the implied two-period zero-coupon yield is equal to 1.1235% which is an annualized yield of 2.2507%. Using the implied one-period and two-period zero-coupon yields, the three-period implied zero-coupon yield solves the equation:

$$100 = \frac{1.25}{(1.010)} + \frac{1.25}{(1.0112535)^2} + \frac{101.25}{(1 + \frac{r_3}{2})^3}.$$

The zero-coupon yield that solves this equation is 1.25195% which is an annualized yield of 2.5039%. The bootstrapping procedure requires that we solve for each subsequent zero-coupon yield and use the results to calculate the next implied zero-coupon yield. Table 2 shows the annual implied zero-coupon yield curve for ten periods.

| | Maturity | Annual Full-Coupon Yield | Annual Implied Zero-Coupon Yield |
|------|----------|--------------------------------|---|
| Bond | t | С | r _t |
| 1 | 1.0 | 2.00% | 2.0000% |
| 2 | 2.0 | 2.25% | 2.2507% |
| 3 | 3.0 | 2.50% | 2.5039% |
| 4 | 4.0 | 3.00% | 3.0148% |
| 5 | 5.0 | 3.40% | 3.4277% |
| 6 | 6.0 | 3.70% | 3.7396% |
| 7 | 7.0 | 4.10% | 4.1631% |
| 8 | 8.0 | 4.45% | 4.5403% |
| 9 | 9.0 | 4.70% | 4.8108% |
| 10 | 10.0 | 5.00% | 5.1442% |

Table 2: Implied Zero-Coupon Yields

For semi-annual coupon bonds, the generalized formula is:

$$P_{n} = \sum_{t=1}^{2n-l} \frac{\frac{C}{2}}{\left(1 + \frac{r_{t}}{2}\right)^{t}} + \frac{100 + \frac{C}{2}}{\left(1 + \frac{r_{n}}{2}\right)^{2n}}.$$

where *P* is the price of the bond, *C* is the annual coupon equal to c(100), and r_t is the zerocoupon yield for a bond maturing in *t* years.

A SIMPLE PRESENT VALUE MODEL FOR VALUING INTEREST RATE SWAPS

In this section, I begin with a simple valuation model for interest rate swaps. The model is based on basic present value relationships. In this model, swap cash flows are defined as a combination of a floating rate bond and fixed rate bond, without principal at maturity.⁴

The value of a fixed rate bond is equal to the present value of the expected future cash flows discounted at the market yield. Similarly, the value of the fixed leg of an interest rate swap, PV_{fixed} , is equal to the present value of the fixed payments:

$$PV_{fixed} = \sum_{t=1}^{mn} \frac{\frac{C}{m}}{\left(1 + \frac{r_t}{m}\right)^t},$$

where C is the fixed annual swap payment, m is the number of payment periods per year, and r_t is the discount rate at time t.

The floating leg of the swap is similar to a floating rate bond that pays no principal. A floating rate bond is always priced at its face value on an interest payment date because the coupon payments adjust to market rates in each time period. The price of a floating rate bond, P, is equal to the present value of its expected cash flows. If we define I_t as the expected floating coupon at time t, then the value of a floating rate bond with face value F is equal to

$$P = \sum_{t=1}^{mn} \frac{\frac{I_t}{m}}{\left(1 + \frac{r_t}{m}\right)^t} + \frac{F}{\left(1 + \frac{r_{mn}}{m}\right)^{mn}}.$$

The only unknowns in the pricing formula are the expected floating rate payments in each time period, I_t . Since interest rate swaps do not require the payment of principal, the first term on the right hand side of the equation is the value of the floating rate leg of the swap. If we rearrange the equation above and replace P with F (since the bond always sell for its face value), the pricing formula can be expressed as:

$$F - \frac{F}{\left(1 + \frac{r_{mn}}{m}\right)^{mn}} = \sum_{t=1}^{mn} \frac{\frac{I_t}{m}}{\left(1 + \frac{r_t}{m}\right)^t}.$$

Therefore, the value of the floating leg of the swap, $PV_{floating}$ is equal to:

⁴ An alternative valuation model includes the principal values at maturity. Since one bond is short and the other is long, the principal amounts cancel.

$$PV_{floating} = F - \frac{F}{\left(1 + \frac{r_{mn}}{m}\right)^{mn}}$$

The value of the swap for the fixed rate payer, V_{fixed} , is:

$$V_{\text{fixed}} = PV_{\text{floating}} - PV_{\text{fixed}},$$

and the value of the swap for the floating rate payer, $V_{floating}$, is: $V_{floating} = PV_{fixed} - PV_{floating}$.

Example: A \$10,000,000 interest rate swap has semiannual payments based on average 3-month LIBOR. The swap matures in 5 years and has a fixed payment rate of 4.50%. The value of the swap for both the fixed and floating rate payers can be calculated using the zero-coupon discount rates in Table 3. DF refers to the discount factor and is calculated as $1/(1 + \frac{r_r}{2})^t$.

| t | r_t | $\frac{r_t}{2}$ | DF | | |
|----|---------|-----------------|--------|--|--|
| 1 | 2.0000% | 1.000% | 0.9901 | | |
| 2 | 2.2507% | 1.125% | 0.9779 | | |
| 3 | 2.5039% | 1.252% | 0.9634 | | |
| 4 | 3.0148% | 1.507% | 0.9419 | | |
| 5 | 3.4277% | 1.714% | 0.9185 | | |
| 6 | 3.7396% | 1.870% | 0.8948 | | |
| 7 | 4.1631% | 2.082% | 0.8657 | | |
| 8 | 4.5403% | 2.270% | 0.8356 | | |
| 9 | 4.8108% | 2.405% | 0.8074 | | |
| 10 | 5.1442% | 2.572% | 0.7757 | | |

Table 3: Discount Rates and Factors

We begin by calculating the value of the fixed leg of the swap. The semiannual payment is (10,000,000(.045)) or (225,000). Therefore, the value of the fixed leg is the present value of the stream of (225,000) payments which is equal to (2,018,484). The valuation of the fixed leg of the swap is summarized in Table 4.

| | | r_t | | | - |
|-------|---------|---------------|--------|------|-------------|
| Т | r_t | $\frac{1}{2}$ | DF | (\$2 | 25,000)(DF) |
| 1 | 2.0000% | 1.000% | 0.9901 | \$ | 222,772 |
| 2 | 2.2507% | 1.125% | 0.9779 | \$ | 220,020 |
| 3 | 2.5039% | 1.252% | 0.9634 | \$ | 216,757 |
| 4 | 3.0148% | 1.507% | 0.9419 | \$ | 211,930 |
| 5 | 3.4277% | 1.714% | 0.9185 | \$ | 206,672 |
| 6 | 3.7396% | 1.870% | 0.8948 | \$ | 201,331 |
| 7 | 4.1631% | 2.082% | 0.8657 | \$ | 194,783 |
| 8 | 4.5403% | 2.270% | 0.8356 | \$ | 188,014 |
| 9 | 4.8108% | 2.405% | 0.8074 | \$ | 181,667 |
| 10 | 5.1442% | 2.572% | 0.7757 | \$ | 174,538 |
| Total | | | | \$ | 2,018,484 |

Table 4: Valuation of the Fixed Leg of a Swap

Using the equation derived above, the value of the floating leg of the swap is: $PV_{floating} = \$10,000,000 - (\$10,000,000)(0.7757) = \$2,242,759.5$

Therefore, the value of the swap to the fixed rate payer is:

 $V_{fixed} =$ \$2,247,759 - \$2,018,484 = \$224,275,

and the value of the swap to the floating rate payer is:

 $V_{floating} =$ \$2,018,484 - \$2,247,759 = -\$224,275.

In this example, interest rates have risen since the issuance of the swap--the swap has a fixed rate of 4.50% and the current five-year market rate is 5.00%. For this reason, the value for the fixed rate payer has increased by \$224,275 and the value for the floating rate payer has decreased by the same amount.

THE TERM STRUCTURE OF INTEREST RATES AND IMPLIED FORWARD RATES

While simple and accurate, the valuation methodology described above has limitations. Complex swaps cannot be valued with a simple model. For complex swap structures we require a more flexible valuation methodology. In this section I develop a swap valuation model based on implied forward rates.

The Pure Expectations Hypothesis

Several hypotheses have been developed to explain how the yield curve conveys information to market participants. The pure expectations hypothesis states that expected future short-term rates are equal to the forward rates implied in the yield curve. One implication of this hypothesis is that the yield curve can be decomposed into a series of

⁵ This value is calculated without rounding the discount factor.

expected future short-term rates that will adjust in such a way that investors receive equivalent expected holding period returns.

Under pure expectations, investors are assumed to be risk-neutral. Since risk-neutral investors apply no risk-related discount to the value of short-term bonds, the shape of the yield curve is driven only by investor expectations. If an upward sloping yield curve prevails, investors expect higher future short-term interest rates, whereas an inverted yield curve implies expectations of lower future short-term rates. This theory implies a flat yield curve when investors expect that short-term rates will remain constant.

The pure expectations hypothesis states that the expected average annual return on a long-term bond is the geometric mean of the expected short-term rates.⁶ For example, the two-period spot rate can be thought of as the one-year spot rate and the one-year rate expected to prevail one year hence. Since expected short-term rates are implied in the yield curve, an investor would be indifferent between holding a 20-year investment, a series of 20 consecutive one-year investments, or two consecutive ten-year investments.

Pure expectations is perhaps the best known and easiest of the theories of the term structure to quantify and apply. For this reason, it is widely used in the capital markets as a pricing convention for interest rate contingent securities. The set of forward rates derived under pure expectations, the implied forward yield curve, is the basis for the valuation of many fixed-income securities.

Implied Forward Rates

Coupon bonds may be viewed as a portfolio of zero-coupon bonds with unique yields, r_t , for each coupon payment received at time t. As such, coupon bonds can be viewed as a series of separate bonds of different overlapping maturities. Consider bond two in the above example. Using zero-coupon yields, the bond can be priced as:

$$100 = \frac{1.125}{(1.0100)} + \frac{101.125}{(1.0112535)^2}$$

The first cash flow is discounted at a yield of 2.0000% for one year and the second cash flow is discounted at a yield of 2.2507% for two years. An alternative view of the second cash flow is that it is invested over two one-year periods. Since we know the yield over the first period, there is an implied yield for the second period that satisfies the following relationship:

$$100 = \frac{1.125}{(1.0100)} + \frac{101.125}{(1.01000)(1+_1 f_2)}$$

The implied yield for the second period, lf_2 , is the forward rate on the bond from period 1 to period 2 and is equal to 1.25085% which is an annualized rate of 2.5017%. The implied

⁶ Empirical evidence suggests that implied forward rates have been a poor predictor of future short-term interest rates. Fama (1975) found that a naïve forecasting method, which used current rates to predict future short-term rates, produced more accurate forecasts than one using implied forward rates.

forward rate is simply the yield earned on a one-period bond from period 1 to period 2 when the investor contracts to invest in the bond today.

Viewed in this way, long-term bonds can be considered a portfolio of a one-period investment at the prevailing spot rate of interest and a series of forward contracts to invest in one-period bonds at rates agreed upon today. The one-period forward interest rates are embedded in the price of long-term bonds and can be calculated from the zero-coupon yield curve using the equation:

$$\left(1+\frac{r_t}{2}\right)^t = \left(1+\frac{r_1}{2}\right)\left(1+\frac{1f_2}{2}\right)\left(1+\frac{2f_3}{2}\right)\dots\left(1+\frac{t-1f_t}{2}\right),$$

where $t_{t-1}f_t$ is the annualized implied forward rate for period t-1 to t and r_t is the annualized implied zero-coupon yield for time t. Solving for $t_{t-1}f_t$, the implied forward rate for period t is be calculated as

$$_{t-1}f_{t} = 2 \left\lfloor \frac{\left(1 + \frac{r_{t}}{2}\right)^{t}}{\left(1 + \frac{r_{1}}{2}\right)\left(1 + \frac{1}{2}f_{2}\right)\left(1 + \frac{2}{2}f_{3}\right)\dots\left(1 + \frac{t-2}{2}f_{t-1}\right)} - 1 \right\rfloor.$$

Alternatively, this calculation can be expressed as:

$$_{t-1}f_{t} = 2 \left| \frac{\left(1 + \frac{r_{t}}{2}\right)^{t}}{\left(1 + \frac{r_{t-1}}{2}\right)^{t-1}} - 1 \right|.$$

Table 5 presents the annualized implied forward yields associated with the implied zerocoupon yields from our example.

| Implied-Forward Yields | | | | | |
|------------------------|--------------|---|--|--|--|
| Bond | Maturit y | Annual Implied Zero- Coupon Yield | Annual Implied Forward Yields | | |
| | t | r_t | $t-1f_t$ | | |
| 1 | 1.0 | 2.0000% | 2.0000% | | |
| 2 | 2.0 | 2.2507% | 2.5017% | | |
| 3 | 3.0 | 2.5039% | 3.0112% | | |
| 4 | 4.0 | 3.0148% | 4.5552% | | |
| 5 | 5.0 | 3.4277% | 5.0880% | | |
| 6 | 6.0 | 3.7396% | 5.3061% | | |
| 7 | 7.0 | 4.1631% | 6.7229% | | |
| 8 | 8.0 | 4.5403% | 7.2003% | | |
| 9 | 9.0 | 4.8108% | 6.9876% | | |
| 10 | 10.0 | 5.1442% | 8.1693% | | |

| Table 5: Implied Zero-Coupon and |
|----------------------------------|
| Implied-Forward Yields |

An alternative swap valuation method uses implied forward rates. The floating rate leg is valued as the present value of expected cash flows using the implied forward rates, $_{t-1}f_1$, to calculate the expected cash flows. Using this method, the value of a swap to the floating rate payer is:

$$V_{floating} = \sum_{t=1}^{mn} \frac{\frac{C}{m}}{\left(1 + \frac{r_t}{m}\right)^t} - \sum_{t=1}^{mn} \frac{\frac{t-1}{m}f_t}{\left(1 + \frac{r_t}{m}\right)^t},$$

and the value of a swap to the fixed rate payer is:

$$V_{fixed} = \sum_{t=1}^{mn} \frac{\frac{t-1}{m} f_{t}}{\left(1 + \frac{r_{t}}{m}\right)^{t}} - \sum_{t=1}^{mn} \frac{\frac{C}{m}}{\left(1 + \frac{r_{t}}{m}\right)^{t}}$$

In the previous example, the fixed rate on the swap is 4.50% and the notional amount of the swap is \$10,000,000. Using the information from Table 2, we can calculate implied forward rates. Table 6 shows the implied forward rates and the present values of the implied forward rates and fixed rate payments. The present value of the floating rate payments is calculated as the sum of the discounted values of the implied forward rates. The present value of the fixed rate payments is the sum of the discounted value of the 2.25% fixed rate payments. The value of the fixed leg of the swap is the sum of the net payments as a percentage of notional principal. The value of the fixed leg of the swap is \$224,275. The same value is calculated using the previous model.⁷

| | $\underline{r_t}$ | $t-1f_t$ | | | | Net Payment |
|-------|-------------------|----------|--------|------------------------|--------------|-------------|
| t | 2 | 2 | DF | PV _{floating} | PV_{fixed} | (Fixed Leg) |
| 1 | 1.000% | 1.000% | 0.9901 | 99,010 | 222,772 | (123,762) |
| 2 | 1.125% | 1.251% | 0.9779 | 122,317 | 220,020 | (97,703) |
| 3 | 1.252% | 1.506% | 0.9634 | 145,046 | 216,757 | (71,711) |
| 4 | 1.507% | 2.278% | 0.9419 | 214,530 | 211,930 | 2,601 |
| 5 | 1.714% | 2.544% | 0.9185 | 233,675 | 206,672 | 27,003 |
| 6 | 1.870% | 2.653% | 0.8948 | 237,398 | 201,331 | 36,067 |
| 7 | 2.082% | 3.361% | 0.8657 | 291,000 | 194,783 | 96,217 |
| 8 | 2.270% | 3.600% | 0.8356 | 300,835 | 188,014 | 112,820 |
| 9 | 2.405% | 3.494% | 0.8074 | 282,092 | 181,667 | 100,425 |
| 10 | 2.572% | 4.085% | 0.7757 | 316,858 | 174,538 | 142,320 |
| Total | | | 8.9710 | 2,242,759 | 2,018,484 | 224,275 |

Table 6: Swap Valuation Using Implied Forward Rates

CALCULATING AN AT-MARKET SWAP RATE

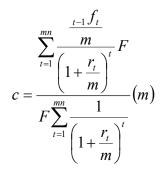
Generally, at the time an interest rate swap is settled, the present value of the expected net payments has a value of zero. Neither party expects to have zero payments in every period. If the yield curve is upward-sloping, the fixed-rate payer expects to make positive swap payments in the early years and receive positive swap payments in the later years. If the yield curve is downward sloping, the fixed-rate payer will expect to receive positive swap payments in the early years and make positive swap payments in the later years. In a flat yield curve environment, the expected future payments for both the fixed and floating rate payers are zero. In any interest rate environment, the fixed rate that makes the present value of the expected net payments equal zero is known as the at-market swap rate.

Recall from above that the value of a swap is based on implied forward rates. When solving for the at-market swap rate, all of the valuation inputs are known except for the fixed swap rate. For an at market swap, the following must hold:

$$\sum_{t=1}^{mn} \frac{\frac{c}{m}F}{\left(1+\frac{r_t}{m}\right)^t} = \sum_{t=1}^{mn} \frac{\frac{t-1}{m}f}{\left(1+\frac{r_t}{m}\right)^t}.$$

Solving the equation for the at-market swap rate, *c*, we get:

⁷ The slight difference is due to rounding.



The at-market swap rate is equal to the sum of the present value of the implied floating rate payments divided by the notional amount times the sum of the discount factors. This amount is multiplied by the number of payment periods per year.

Example: Using the data from Table 6, we calculate the at-market swap rate. The sum of the present value of the floating rate payments is 2,242,749. The sum of the discount factors is 8.9710; therefore the at-market swap rate is [2,242,749 / (8.9710*10,000,000)](2) = 500,000. Therefore the at-market swap rate is approximately equal to 5.00%.

Table 7 summarizes the expected cash flows on the floating and fixed rate legs of the swap using the 5.00% at-market swap rate. This proves the accuracy of the at-market swap rate since the sum of the net payments (at present value) expected on the swap at the at-market swap rate is zero. This holds since PV_{fixed} and $PV_{floating}$ are both equal to 2,242,749.

| | | | Nat Daymout |
|-------|------------------------|---------------------|-------------------|
| T | DI | | Net Payment |
| Т | PV _{floating} | PV _{fixed} | (to Floating Leg) |
| 1 | 99,010 | 247,525 | (148,515) |
| 2 | 122,317 | 244,467 | (122,150) |
| 3 | 145,046 | 240,841 | (95,795) |
| 4 | 214,530 | 235,477 | (20,947) |
| 5 | 233,675 | 229,635 | 4,039 |
| 6 | 237,398 | 223,701 | 13,697 |
| 7 | 291,000 | 216,426 | 74,574 |
| 8 | 300,835 | 208,905 | 91,930 |
| 9 | 282,092 | 201,852 | 80,239 |
| 10 | 316,858 | 193,931 | 122,927 |
| Total | 2,242,759 | 2,242,759 | 0 |

 Table 7: Calculation of At-market Swap Rate

SUMMARY

In this paper, I present two simple models as an introduction to interest rate swap pricing for students. The models are meant to facilitate the understanding of how interest rate swaps are structured and how interest rate movements affect their value. I also show a simple computation of the at-market swap rate which further emphasizes these ideas. All three examples are meant to build a basic understanding of interest rate swaps.

Swaps are an important hedging tool. As the swap market continues to grow it becomes an increasingly vital part of the finance curriculum. The models presented in this paper are well-suited for use in a financial modeling class.

Swap valuation in practice is much more detailed and require many inputs from the capital markets. Swap traders use sophisticated models that generate real-time swap values based on live data feeds. However, the models outlined in this paper are accurate and provide insight into swap pricing and the mechanics of the swap market.

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