

A Simple Analytical Examination of Fama and French (1992) Empirical Results

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ABSTRACT

Explanatory variables popularly employed for empirical studies of CAPM can be regarded as different ways to scale stock prices. Since the return is a change in prices, one may suspect that anomalies documented may be a statistical artifact. In this paper, a simple analytical examination is conducted with a cross-sectional regression of a projected stock return on the previous period stock price. The stock price or stock return is assumed to follow AR(1). It can be shown that the regression coefficient of any price scaled variable is biased and tends to produce Fama and French (1992) results.

INTRODUCTION

According to Fama and French (1992) and Keim (1988), independent variables popularly used for empirical studies of Capital Asset Pricing Model (CAPM), such as firm size, book-to-market equity (B/M) ratio, P/E ratio, and leverage can be understood as price scaled variables. Since the return is a change in prices, one may suspect that the anomalous empirical results obtained by Fama and French (1992) and others (e.g., Banz (1981), and Stattman (1980)) in the extant literature, explaining the cross-sectional variation in stock returns with those variables, may be a statistical artifact, partially due to the time-series effect of the price change on the cross-section stock price relation.

In this paper, a simple analytical examination is conducted with a cross-sectional regression of a projected stock return on the previous period stock price. The stock price or return is assumed to follow a first order autoregressive process. In the event that the stock price follows AR(1) as documented by Summers (1986) and has a long decaying component, a time-series movement of stock price indeed has an impact on the following period stock price in such way that the slope coefficient of the cross-sectional regression yields the anomalous results evidenced in the literature since Fama and French (1992). On the other hand, if the stock price has a non-stationary stochastic process as its return rather follows AR(1), the cross-sectional regression slope coefficient shows no clear sign of anomaly, especially if the return is measured over a long horizon period.

While it is not clear as to how long is “long” and how much of the stock price movement is determined by the temporary and permanent components to produce the anomalous results, our examination demonstrates that the regression coefficient of the previous period price, functioning

as the explanatory variable, may be biased and tends to produce the popular anomalous results documented. One may consider this result as consistent with the small firm effect and with the B/M effect.

While the results of a simple one-period cross-sectional regression analysis are examined here, the results obtained by employing methods often used for empirical asset pricing models, including pairing or the two-step approach of Fama and McBeth (1973) as discussed in Fama and French (2008), are expected to be little different..

Although our analysis casts doubt on the anomalous empirical results of CAPM as being spurious, our analytical work does not necessarily support CAPM by refuting entirely the results of empirical studies reporting the anomalies. We only recommend further investigation of the validity of CAPM by examining whether the beta is the only statistically significant variable explaining the cross-sectional difference in the projected return.

It is also imperative to further explore the structure of serial correlation in stock price and to study how much this serial correlation affects the cross-section variation of projected stock returns. For this sort of study, it may be useful to decompose each price scaled variable into its price component and another unique component. This way, one can learn more about the source of any anomaly that may exist. Empirical studies of this nature, however, are left for a future study.

A SIMPLE ANALYTICAL EXAMPLE

Issues such as misspecification and “errors-in-variables,” often raised in the empirical studies of asset pricing models are ignored in this study to focus on the relation between the projected return and market and the previous period stock price as captured in the cross-sectional regression equation below:

$$P_{it} - P_{it-1} = \alpha_0 + \alpha_1 r_{it} + \alpha_2 P_{it-1} + \epsilon_{it}, i=1, \dots, n \text{ for a given } t$$

where r_{it} (possibly, time varying) captures the systematic risk and the price variables are log-valued.¹ The price variable is also adjusted for price impact factors such as stock splits, stock and cash dividends, and stock repurchases so that the price changes captures total holding period rate of return. The error term, ϵ_{it} , is assumed to be independent of P_{it-1} .

If the stock price follows a first order autoregressive process (AR(1)),²

$$P_{it} = P_{it-1} + \epsilon_{it}$$

where ϵ_{is} and ϵ_{jt} are serially independent of themselves, independent of each other for all i, j, s , and t , and independent of P_{kt-1} , for any k .

This slowly decaying stationary price movement can be justified according to Summers (1986) as he argues for long price swings with ρ being close to 1 in an inefficient market. Alternatively, this AR(1) in the stock price may be due to a time-varying equilibrium expected returns by rational investors (Fama and French 1988). Note that the AR(1) coefficient, ρ , is the same for all stocks for the sake of simplicity.

It can be shown that the probability limit of β will be zero in this setup, while that of the cross-sectional regression coefficient of P_{it-1} will become $\rho - 1$ as n , the number of stocks, increases to infinite. For a stationary AR(1) with $|\rho| < 1$, $\rho - 1$ is expected to be negative, producing a spurious small firm effect, assuming that the number of shares outstanding, which is pretty steady over time, is not really an issue in the cross-sectional variation of returns. For a price scale variable like the B/M ratio, the price component will become $-P_{it-1}$ (note that all the price variables are log-valued) and the coefficient of B/M ratio is expected to be positive, producing a spurious positive B/M effect on the cross-section variation of stock returns as long as the cross-sectional variation of the book value is, over a “long” period time, is eventually dominated by that of the market value.

The intuition behind the negative relationship between the projected return, $P_{it} - P_{it-1}$, and the explanatory variable, P_{it-1} , for a stationary AR(1) is that the price has a regressive process with the regression coefficient, ρ , being less than 1.0, and the previous period price is the basis for the determination of the projected return. While the time-series effect of the price movement on the cross-section regression is expected to be present for a different description of the serial correlation, this spurious result may cease to exist for a random walk process, as the previous price does not have any explanatory power over the future returns.

If the stock return rather than the stock price, $r_{it} = P_{it} - P_{it-1}$, follows the AR(1) process as in Fama and French (1988), the stock price has a non-stationary component and the stock i 's return can be described like $r_{it} = r_{i,t-1} + \eta_{it}$, where $\eta_{it} \sim \text{IID}(0, \sigma^2)$.³ In a special case where $\sigma = 0$, this process becomes a random walk. According to Lo (2004), ρ value ranges from 0.1 to 0.4 for the most part of his 1871~2003 study period. Now, we need to examine the probability limit of the cross-sectional regression coefficient, $\text{Cov}(r_t, P_{i,t-1})/\text{Var}(P_{i,t-1})$, as the number of stocks goes to infinite. For simplicity, subscript i for stock i is suppressed. Given $r_t = r_{t-1} + \eta_t$, it can be shown that

$$r_t = \rho^{t-1}(P_1 - P_0) + \rho^{t-2}\eta_2 + \rho^{t-3}\eta_3 + \dots + \rho^2\eta_{t-2} + \rho\eta_{t-1} + \eta_t.$$

In addition,

$$P_{t-1} = P_1 + \frac{\rho(\rho^{t-2} - 1)}{\rho - 1}(P_1 - P_0) + \frac{\eta_2(\rho^{t-2} - 1)}{\rho - 1} + \frac{\eta_3(\rho^{t-3} - 1)}{\rho - 1} + \dots + \frac{\eta_{t-3}(\rho^3 - 1)}{\rho - 1} + \eta_{t-2}(1 + \rho) + \eta_{t-1}$$

Assuming that the initial public offering price P_0 is not random,

$$Cov(P_t - P_{t-1}, P_{t-1}) = \frac{\rho^t(\rho^{t-2} - 1)}{\rho - 1} Var(P_1) + \frac{\rho}{(\rho - 1)^2} \left[\frac{\rho(\rho^{2(t-2)} - 1)}{\rho + 1} - \rho^{t-2} + 1 \right] \sigma^2$$

and

$$Var(P_{t-1}) = \left(\frac{\rho^{t-1} - 1}{\rho - 1} \right)^2 Var(P_1) + \left[\frac{\rho^2(\rho^{2(t-2)} - 1)}{\rho^2 - 1} - \frac{2\rho(\rho^{t-2} - 1)}{\rho - 1} + t - 2 \right] \frac{\sigma^2}{(\rho - 1)^2}$$

For a sufficiently large t , the numerator covariance approaches $\rho^t / [(\rho - 1)^2(\rho + 1)] \sigma^2$, while the denominator variance linearly increases $[(t-2)/(\rho - 1)^2] \sigma^2$. This means that the slope coefficient of P_{t-1} does not produce anomalies we have seen in the previous example of AR(1) in stock price as it approaches 0.0.

Instead of combining two types of price movement components, one being AR(1) in stock prices and the other being a simple non-stationary process as in Fama and French (1988), we have examined how each type of price movement has an impact on the estimation of the cross-sectional regression coefficient of a popular price scaled variables. In the case of AR(1) in stock prices, we can confirm that the cross-sectional regression effects of previous period stock prices on the subsequent period returns could generate the popular anomalous results involving price scaled variables. On the other hand, these anomalies may not be present if the stock price follows the simple non-stationary process with AR(1) in returns.

FURTHER DISCUSSION

Further examination of this problem can occur in a variety of ways. Empirical results of the beta-pricing model may be subject to a choice of estimation methods. Instead of a simple cross-sectional regression à la Fama-McBeth (1973), one may consider the sorting approach used in Fama-French (1992). A time-series and cross-sectional joint estimation in a maximum likelihood estimation or seemingly unrelated estimation can be used as well. Rather than using an individual stock, one can use portfolios to reduce measurement and estimation errors. According to Fama and French (2008) and Shanken (1992), analytical examination results of the time-series effect on the cross-section pricing estimation as shown in our example are expected to prevail regardless of a choice of an estimation method, yielding spurious anomalies.

A more challenging question is to have additional price scaled variables in the cross-sectional regression of projected return on these variables. Unlike a simple regression with market return and a previous stock price as explanatory variables in our example, one may expect a strong multicollinearity problem to arise in this multivariate regression as the additional price scaled variables have a common price component, resulting in low t-value for each regression coefficient even if these variables collectively have good explanatory power collectively. While it remains an empirical issue, each variable may have a distinctive unique component that cross-sectionally differentiates it from other variables. If that is the case, then one may expect that the

empirical results would not be markedly different from those already obtained in the literature since Fama and French (1992). To the extent that each popularly employed price scaled variable has such a distinctive element to produce a statistically significant coefficient in a multivariate regression, the findings of an empirical anomaly in the extant literature cannot be entirely spurious. However, the degree of the time-series effect of the price attributable to the anomaly still needs to be assessed. Furthermore, it is necessary to identify the exact source of such an anomaly if the conventional price scaled variables are to be conditioned to strip away the common price component.

Jegadeesh and Titman (1993) also explore, in the context of market timing, a relationship between the cross-sectional pricing relationship and the time-series movement of the price. Their argument is, however, that the different stock return movement over time between the successful and unsuccessful firms might be attributable to cross-sectional risk disparity among stocks. In other words, their focus is on the misspecification of the pricing model and on the lack of consideration for a relevant risk factor on the anomalous time-series behavior of stock prices, an opposite perspective of this paper. In a recent paper, Fama and French (2007) study how migration of firms across size and value portfolios contributes to the size and value premiums in average stock returns. Their study is also exploring how a time-series movement of stock prices has an impact on the cross-sectional variation of stock returns. It needs to be pointed out, however, this manifestation of the time-series effect in an empirical study of the cross-sectional pricing relation may not necessarily capture the risk-return relationship any asset pricing model tries to delineate. As Fama and French (1996) explained, the CAPM may be still wanted, whether the explanatory power of beta is limited or not in empirical studies. Likewise, one may be careful about taking a significant coefficient of any “random” variable as being indicative of an anomaly as the statistical significance may be an artifact which may have no bearing on the true cross-sectional pricing relation.

CONCLUSIONS

In this paper, a simple analytical examination is conducted with a cross-sectional regression of a projected stock return on market and the previous period stock price, which is assumed to have either a first order autoregressive process or a random walk process. This example demonstrates that CAPM anomalies involving the price scaled variables may be due to the time series effect of price changes in the case the stock price follows AR(1).

Despite the fact that the anomalous empirical results of CAPM may be spurious, we still recommend further investigation into the validity of CAPM by examining whether the beta is the only variable which can statistically significantly explain the cross-sectional difference in the projected return.

It is also imperative to empirically explore the exact structure of the serial correlation in stock price and to explore how much this serial correlation affects the cross-section variation of projected stock returns. For this sort of study, it may be useful to decompose each price scaled variable into its price component and another unique component. This way, one can learn more

about the source of any anomaly, if it exists. Empirical studies of this nature remain as a future research issue to be addressed.

ENDNOTES

¹ Asset pricing theory suggests factor loading as the better choice in a cross section specification instead of firm characteristics (e.g., B/M, size) and the literature since Fama and French (1992) has tended to use related factor loadings. For the sake of simplicity and to the extent that the results Fama and French obtained subsequently are much the same as their 1992 paper, we use P_{it-1} , expecting that using the factor loadings would have the same element of the statistical artifact we show analytically in this paper.

² For simplicity, autocorrelation coefficients are assumed to be fixed instead of being firm-specific. The main results should remain the same in a complex varying coefficient model, except that the cross-sectional pricing regression coefficient will become an "average" of the individual coefficients.

³ Instead of combining the stationary and non-stationary components together in one equation describing the stock price movement process as in Fama and French (1988), we are dealing with them separately for simplicity. Since we are dealing with each process individually, we elect to use the same popular notation, ρ , repeatedly.

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